Harvard-Smithsonian Center for Astrophysics

Precision Astronomy Group

MEMORANDUM

Date: 10 December 1998 TM98-07
To: Distribution
From: R.D. Reasenberg
Subject: Repetition of observations by FAME in contiguous and near-contiguous rotations.

I. Introduction.

As the FAME instrument rotates and its rotation axis precesses around the Sun direction, the field of view that it sweeps out overlaps from one rotation to another. The overlap is the smallest (but still substantial, with present nominals) where the view direction is 90 deg from the Sun direction; it is the largest approximately where the view direction is closest or furthest from the Sun direction. The overlap is believed to be important during the spiral reduction phase of the analysis, where it contributes to the "rigidity" or "cohesion" of the spiral. This rigidity plays a role in the immunity of the mission catalog from regional bias and the potential to determine the center of a group of neighboring stars to high accuracy by averaging their determined positions. A low level of rigidity can increase the uncertainty of the estimate of the position of a single star.

Recent work by Chandler and Reasenberg (paper in preparation) has suggested a need to understand the overlap in more detail. The following analysis responds to that need.

II. Analysis.

Figure 1 shows the spherical geometry of the problem, with the Sun at S. The reference plane is perpendicular to the Sun direction, and XYS is a right-handed system of orthogonal axes, with S and X in the ecliptic. A rotation around S (i.e., CW around the S direction) by the precession angle, v = XOA, results in a coordinate system ABS, where A is the ascending node of the observation plane on the reference plane, and the observation plane is perpendicular to the spacecraft rotation vector, R (in the precessing frame). The rotation of the spacecraft is described (in the Sun-oriented frame) by the Eulerian angles v, ξ, and φ, where ξ = SOR and φ is measured (CW around the R direction) from A to P, the mean direction of the instrument's view ports. The variation of the Sun direction is neglected in the calculation.

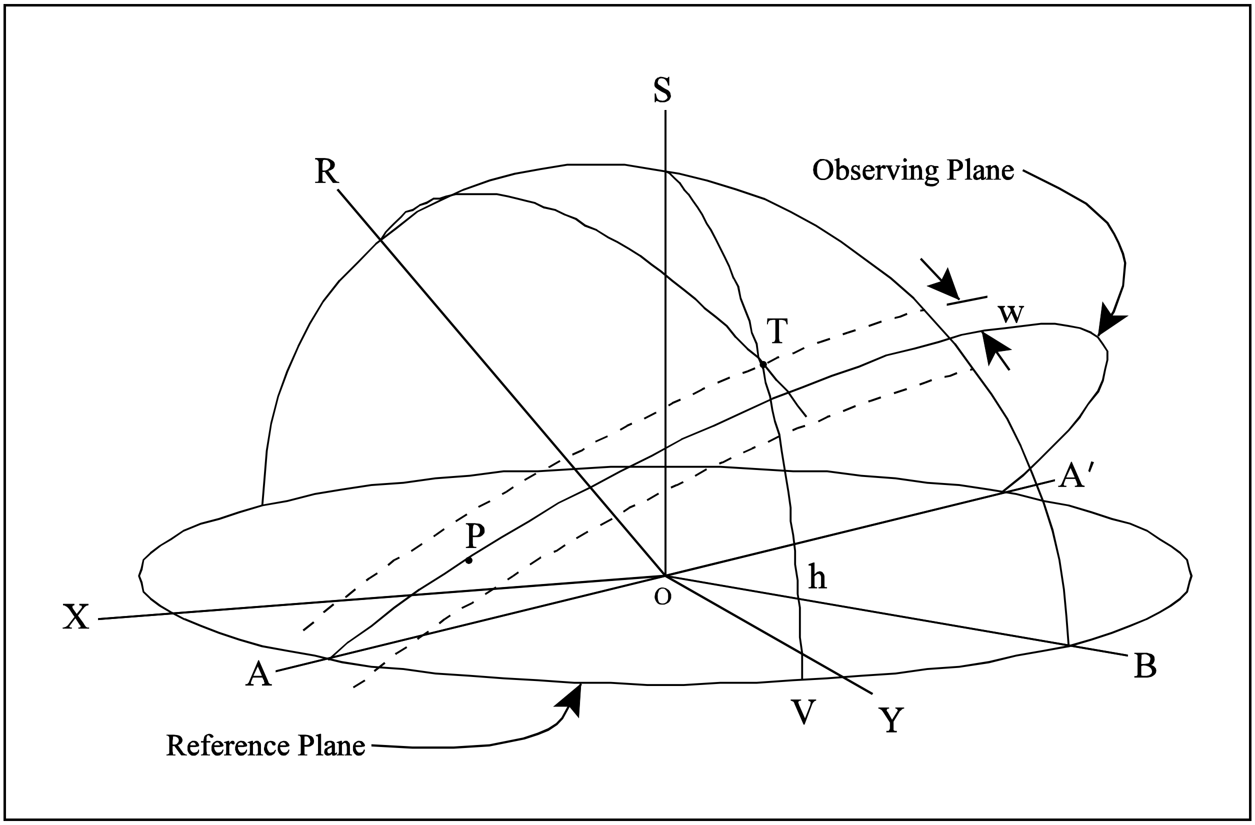

Figure 1. The direction of view is 15 deg above the reference plane and 70 deg from the x axis.

Consider a target star at T. In the XYS system, the target has coordinates (g, h), where g is measured in the reference plane from X (toward Y) and h is measured from the reference plane (toward the Sun). In the ABS system, the target has coordinates (u, h), where u is measured in the reference plane from A (toward B), and g = u + v. We ask whether (and for what values of v) the star is within the observation band of width 2w centered on the observing plane, i.e., within the region swept out by the field of view. For stars that can be within the observation band (for a particular Sun direction), we will find ρ, the size of the range of v for which the star will be seen in every rotation. We need to consider two cases separately: $|h| < |\xi - w|$ and $|h| > |\xi - w|$.

CASE I, $|h| < |\xi - w|$. We address this problem by finding (for fixed h) the values of u for which T is at the upper and lower bound of the observing band, u_- and u_+ respectively. (In Fig. 1, T is shown at the upper bound.) The value of ρ follows trivially. Considering spherical triangle RST, we note that

$$
\begin{aligned}
\square\,\square &= \xi \\
ST &= \pi/2 - h \\
RT &= \pi/2 \pm w \\
\angle S &= \pi/2 + u_{\pm}
\end{aligned}
\qquad \mathbf{(1)}
$$

For that spherical triangle,

$$\cos(\pi/2 \pm w) = \cos(\xi)\cos(\pi/2 - h) + \sin(\xi)\sin(\pi/2 - h)\cos(\pi/2 + u_{\pm}) \tag{2}$$

$$\sin(u_{\pm}) = \frac{\cos(\xi)\sin(h) \pm \sin(w)}{\sin(\xi)\cos(h)} \tag{3}$$

Note that for both u_+ and u_- there are a pair of solutions that are symmetric around B. Note also that u_+ corresponds to the larger of the two options for RT, which puts T on the lower side of the band. It is convenient to define

$$u_0 \equiv \left(\frac{u_+ + u_-}{2} \right) \tag{4}$$
$$\Delta u \equiv u_+ - u_-$$

Then, to a useful approximation, we may write

$$u_0 \approx \operatorname{asin}\left(\frac{\tan(h)}{\tan(\xi)} \right) \tag{5}$$

$$\square\square \left(\frac{\Delta u}{2} \right) \approx \frac{\sin(w)}{\sin(\xi)\cos(h)\cos(u_0)} \tag{6}$$

The size of the range of v for which the star will be seen in every rotation is $\rho = |\Delta u|$. Finally, for small h we find

$$\Delta u \approx \frac{2w}{\sin(\xi)} \tag{7}$$

which has been known for a long time by members of the FAME project.

CASE II, $|h| > |\xi - w|$. We address this problem by finding the values of u for which T is at the upper bound of the observing band (i.e., furthest from the reference plane.) By setting the right hand expression of Eq. 3 equal to ±1, we can easily show that $|h| \leq \xi + w$ for a star that can be observed, which is also easily seen from the geometry in Fig. 1. The general solution for u follows directly from Eq. 3.

$$\sin(u) = \frac{\cos(\xi)\sin(h) - \sin(w)}{\sin(\xi)\cos(h)} \quad \textbf{(8)}$$

Again, there is a pair of solutions for u and they are symmetric about B; ρ is the absolute difference between these values. The largest range of v for which a target is in the observing band, ρ_{max}, is at $h = \xi - w$. For small w, it can be shown that

$$\rho_{max} \approx 2\sqrt{\frac{w}{\sin(\xi)\cos(\xi)}} \quad \textbf{(9)}$$

For the present nominal design, $\xi = 45$ deg and $w = 1.1$ deg, the above expression yields $\rho_{max} \approx$ 44.91 deg. (The exact calculation yields 44.77 deg.) For the nominal precession period of ten days and rotation period of 20 minutes, this corresponds to 1.25 days or 90 spacecraft rotations.

III. Discussion

Figure 2 shows ρ, the size of the range of precession angle (v, in degrees) over which a star is observed in successive spacecraft rotations. To convert to time in hours, multiply by 2/3 (based on the mission nominal rate of precession -- one per ten days). The discontinuity in the plot corresponds to the transition between the two cases. The size of the precession range varies slowly for small h, doubling between h = 0 (where it is 3.11 deg) and h = 38 deg., but then increasing seven fold (to 22.4 deg from the value at h = 0) by the transition angle of 43.9 deg. For CASE I, there are two sets of contiguous rotations for observing the target star during a single cycle of precession. The gap between these sets is also shown in Fig. 2, and can be seen to go to zero at the transition between cases.

The analysis has been performed neglecting the Sun's motion of ≈1 deg per day. From a casual inspection of the full problem, I conclude that for present purposes, the corrections are not important. They will, of course, be essential for the analysis of real data.

I have examined curves like those in Fig 2, but with w increased to 1.5 deg. The change is as expected. The observation span increases in proportion to w at h = 0, and in proportion to $w^{1/2}$ at the transition angle, which is decreased by the increase of w. The anticipated small changes (about two degrees over the precession cycle -- Tech Memo in preparation by Reasenberg) in the Sun-spin angle, ξ, will do little to change the curves. The most dramatic effect is to shift the transition angle, but this should not materially affect the rigidity of the spiral solution.

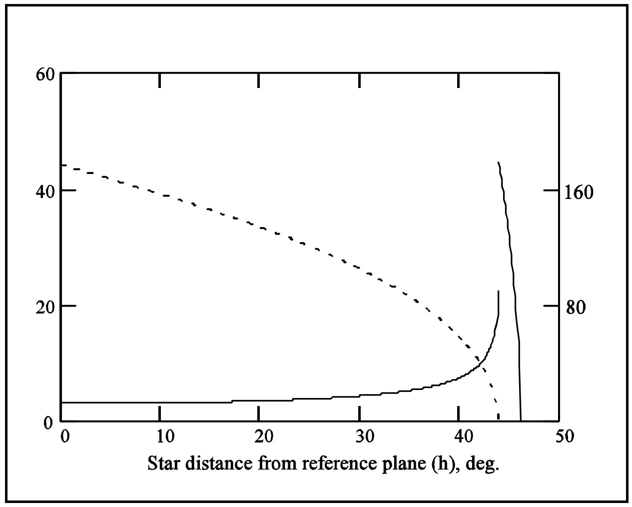

Figure 2. Observation range, ρ (solid line, scale on left in deg), and gap between observation ranges (dashed line, scale on right in deg) as a function of h, the distance of the star from the reference plane. The left-hand and right-hand portions of the observation span curve correspond to Case I and Case II, respectively.

IV. Comparison of the coordinate system used with that of Hipparcos.

The coordinate systems used in the present analysis (Fig. 1) are similar to those defined in the Hipparcos project. (For Hipparcos coordinates, see: ESA SP-1111, Vol 3, Fig. 1.10 on page 14; or ESA SP-1200, Vol 2, Fig. 8.1 on page 144.) The above analysis started as a quick calculation (using Hipparcos-like geometry and nomenclature), and grew into the present Tech Memo. Only after the fact did I consider the connections given here. The present work starts from a reference plane perpendicular to the Sun direction (and thus perpendicular to the ecliptic) and uses Eulerian rotations to get to the spacecraft frame. The Hipparcos analysis starts with an ecliptic frame, and defines the spacecraft spin vector by a pair of (azimuth-zenith) angles; the rotation phase, within the precessing frame, is with respect to the plane containing the Sun direction (and the spin vector of the precessing frame.) A translation between the systems is given in the table on the next page. The geometry of Fig. 1 was chosen to simplify the present problem. It is not necessarily (in fact, unlikely) the geometry to use for the analysis of the FAME data.

Connection between notation used for the coordinate system of Fig. 1 and the coordinate system used in the analysis of the Hipparcos data.	
This work	Hipparcos (See either of the above references.)
R	Z
S	λ_s
P	X
Eulerian angle φ	$\Omega = \varphi + 3\pi/2$
Eulerian angle v	$\nu = v + 3\pi/2$

V. Acknowledgments

I thank J.D. Phillips for reviewing this memorandum in draft form.

VI. Distribution

FAME web site *via* S. Horner.

SAO internal:

R.W. Babcock
J.F. Chandler
J.D. Phillips
R.D. Reasenberg
I.I. Shapiro