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Subject: GAMES sensitivity to complexity of rotation

I. INTRODUCTION

Reasenberg (1997) discussed a scheme for analyzing in three stages the data from a GAMES mission based on a nominal spacecraft design. The present study pertains to the first stage of analysis (modeling the spacecraft rotation from "batches" of astrometric data) and addresses the question of whether frequent rotation corrections (e.g., firing of a gas jet or any other torsional impulse) of the GAMES spacecraft would seriously degrade the astrometric output of a mission. At issue is whether the swath of sky seen by the instrument during a "batch interval" (of order one day) can be securely tied together in the face of frequent rotation corrections. The remainder of this memorandum describes a simulated implementation of the first part of the data analysis (Sections II - IV), presents the results from a series of simulations (Section V), and draws conclusions about spacecraft design considerations (Section VI). In particular, we conclude that the scientific output of the mission would be significantly higher if frequent attitude correction events could be avoided.

II. SPACECRAFT OPERATIONAL DESCRIPTION

In a real mission, we expect that there would be a period of latency after each rotation correction, when no useful observations could be made, since the action of attitude control gas jets is likely to have a component that changes the spin rate, thus requiring the data immediately after each attitude control event to be used to re-estimate the spin rate. However, for simplicity, the present study makes no allowance for such a latency time. Thus, each star is observed four times per rotation (once in each field of view by each of the two rows of detectors) as long as it lies close enough to the plane normal to the spacecraft's rotation axis. The total width of the field of view is 0.75 deg in all the cases we studied. We have assumed a uniform uncertainty of 0.35 milliarcsec (mas) in the scan direction and 3.5 mas in the cross-scan direction for each observation. In most of these simulations, rotation corrections occur at an average rate of six per rotation. In the terminology of this memorandum, the average length of a "rotation span" is one sixth of the rotation period.

In this study, the spacecraft rotation is modeled as a hierarchy of three motions: (1) the relatively rapid rotation about the spacecraft's nominal spin axis, (2) the somewhat slower precession of that axis about the Sun-spacecraft line, and (3) the annual revolution of that line about the normal to the ecliptic. The third is treated as a known, constant, circular motion, but the other two are treated as time polynomials with adjustable coefficients for the three Euler angles relating the spacecraft frame to the slowly rotating Sun-oriented frame. These three angles are (1) the rotational phase ϕ, (2) the azimuth α (about the Sun direction) of the nominal spin axis, and (3) the elevation δ (positive toward the Sun) of the nominal spin axis.

In the absence of nutations, and in the presence of a steady torque due to (Sun) light pressure, the number of coefficients to be estimated would be 2 + 2 + 1 for the three angles ϕ, α, and δ, respectively (i.e., an initial phase, phase rate, initial precession angle, precession rate, and constant elevation). However, in a system undergoing occasional rotation corrections, a more realistic description might include some higher-order coefficients, as well as a fresh determination of all the coefficients after each correction event. In other words, a separate set of coefficients is estimated for each rotation span. For this reason, the term "rotation break" is sometimes used in referring to the correction events.

In the present study, the span lengths are chosen randomly in a Gaussian distribution with a mean of 20 min and a standard deviation of 5 min. However, the last span is cut short, if necessary, to make the total run length equal to the nominal batch interval (12 hr). In all runs, the nominal rotation period and axis elevation angle are 2 hr and 45 deg, respectively. The study covers two values of the precession rate: 6 and 15 deg/d. At 6 deg/d, the observation bands have a minimum overlap of about 50% from one rotation to the next; at 15 deg/d, there are gaps in the sky coverage.

In addition to the parameters representing spacecraft orientation, the model includes four adjustable parameters that describe the observing geometry: (1) the opening angle between the two fields of view in the scan direction, (2) the scan-direction angular offset between the look angles of the two rows of detectors, and (3 and 4) the cross-scan offsets of the two fields of view from the body-fixed equator. These angles are treated as constants to be estimated from the data. The nominal values used in this series of trials are 45, 0.1, 0, and 0 deg, respectively.

III. STAR OBSERVATIONS

In this study, the sky is populated by stars thrown randomly with a uniform probability density of three per square degree of sky, corresponding roughly to the mean density of stars between mag 8 and mag 10 (Allen 1976, p 243). The model includes two adjustable position parameters for each star observed in the course of a batch interval, which is twelve hours in the present study. The number of observations made during a batch interval depends on the length of the batch interval, the average density of stars, the rotation rate, and the angular width of sky swept out by the detector array, which is 0.75 deg in this study. However, the number of distinct stars seen (and thus the total number of parameters to estimate) depends also on the precession rate, which takes on two trial values in these runs. Since the same Monte Carlo sky is used for all runs in this series, there are only two different numbers of observed stars: 2098 for the cases with 6 deg/d of precession and 3911 for the cases with 15 deg/d. Due to the demands of the data analysis (see Section IV), the observations made in a batch interval must be sorted by star, rather than handled directly in time order.

IV. DATA REDUCTION

A brute-force approach to estimating all these parameters via least-squares techniques would require enormous amounts of computer time. In the case of precession at 15 deg/d, the full information matrix for each instance is about 8000×8000, and the inversion would take about 19 hours of computer time on a Sun Ultra-Enterprise (R) computer. However, the matrix is only sparsely filled, and the processing time is much shortened by pre-reducing (Reasenberg 1975) the matrix in batches of observations from one star at a time (see Reasenberg 1997, p 4). Although the prereduction effort is proportional to the (large) number of stars observed, it takes no more than about ten minutes of computer time per case in these runs, and then the matrix inversion involves at most a few hundred parameters and requires at most a minute of computer time for each case. The results presented in this memorandum thus represent only about two weeks of computer time.

To avoid numerical problems in certain low-probability situations, three kinds of constraints have been applied. First, each rotation coefficient is assumed to be known *a priori* with an uncertainty that gives a peak amplitude of 1000 arcsec over a span. This constraint makes little difference in the results, however, since the solutions generally give uncertainties many orders smaller. Second, the distribution of span lengths is forced into the range 5 - 35 min (i.e., within three standard deviations of the mean) by the expedient of discarding any value outside that range and replacing it by a newly chosen random value. (If necessary, the replacement is replaced in turn, but that is a rare event. For a Gaussian distribution, only about one value in 370 must be discarded.) The very last span may, of course, be shorter than this limit, but the potential numerical problems of a very short span are avoided by the *a priori* constraints. (In processing real data, such short spans would not be created artificially, as we have done here to make different sets of results intercomparable.) Third, the star positions are assumed to be known *a priori* with an uncertainty of 10 mas.

V. RESULTS

The figure of merit used in this study is a measure of the cohesion of the rotation model. It is formed by selecting an evenly spaced grid of epochs covering the entire (twelve-hour) batch interval, with successive epochs separated by 1/100 of that time, and then calculating the covariance for the modeled difference in rotation phase ϕ between pairs of epochs. The logarithms of these covariances are averaged for all possible pairs with lags from 0.11 to 0.50 of the batch interval, and the result is expressed in the form of rms angular uncertainty for that run. (For values lying within a narrow range, the geometric and arithmetic means are practically the same. For values over a wide range, the geometric mean, i.e., averaging the logarithms, gives a more representative result.) To smooth out any dependence on the exact lengths of the rotation spans, sixteen separate runs, with independently chosen sets of rotation breaks (but the same set of stars), are averaged together. In the composite averages, each pair's covariance from each run is included with equal weight. These results are shown in Table 1.

To see why the composite average described above makes a reasonable figure of merit, it is necessary to examine a variety of cases in some detail. In particular,

Table 1. Mean log uncertainty in ϕ (arcsec) as a function of model complexity

	Frequent rotation breaks				No rotation breaks		
	6 deg/d	15 deg/d			6 deg/d	15 deg/d	precession rate
ϕ coeffs	1	1	2	3	1	3	δ coefficients
2	−3.124	−2.994	−2.805	−2.719			
3	−3.123	−2.994	−2.804	−2.718	−4.107	−3.832	
4	−3.123	−2.993	−2.803	−2.717	−4.106	−3.831	
6	−3.122	−2.993	−2.801	−2.714	−4.105	−3.830	
8	−3.110	−2.987	−2.798	−2.711	−4.103	−3.829	
10	−3.075	−2.975	−2.791	−2.706	−4.101	−3.828	
12	−3.071	−2.973	−2.790	−2.704	−4.099	−3.827	
15	−3.056	−2.965	−2.785	−2.700	−4.096	−3.825	
20	−3.040	−2.956	−2.778	−2.694	−4.088	−3.822	
50					−4.066	−3.811	

it is helpful to look at the average uncertainty for each lag separately, i.e., to take averages over all possible grid pairs with a given lag and over a group of 16 runs, but without summing over all the lags. In this way, we may see the structure of the ϕ model's cohesion. Clearly, the averaging over 16 runs is helpful for this purpose, since the irregularities due to uneven rotation spans are smoothed out, and only the "real structure" survives. Figure 1 shows the first of many cases, and is typical of all the cases involving frequent rotation corrections. Figure 2 is similarly typical of the cases with only a single rotation span. There is some structure in both figures, though not much. The first feature is a relatively deep dip near zero lag, which is essentially an average over the whole batch interval. The zero-lag dip is much less prominent in Figure 1, but is present nonetheless. Not surprisingly, there are also dips in the uncertainty at lags corresponding to multiples of the rotation period, but not quite so deep (and not present at all in Figure 2). The same star being observed on successive passes will help to tie together the coefficients of the rotation models for spans separated by a whole number of rotations (but this effect vanishes when there is only one span). The amplitude of these dips in uncertainty can be more than the factor of about 1.5 seen in Figure 1, but is always less than a factor of 3 in the cases studied. There are only faint traces of dips at intervals corresponding to the opening angle between fields of view, though such dips begin to show plainly in some of the cases with large numbers of coefficients to be estimated. The effect is largely suppressed by the smoothing implicit in the limited numbers of coefficients for the rotation models. In any event, the motivation for selecting lags 0.11 to 0.50 is threefold: (1) to avoid the deep dip near zero lag, (2) to include as much in the average as possible, and (3) to include only lags that permit sampling the entire batch interval.

Table 1 shows the results for six different sets of runs, distinguished mainly by varying the number of coefficients in the model of ϕ, but also by varying the spacecraft precession rate, the number of coefficients for δ, and the frequency of rotation corrections. Naturally, increasing the number of parameters to be estimated (whether rotation coefficients or star coordinates) degrades the figure of merit, but the variations seen in Table 1 are all relatively minor, except between the runs on the left (with frequent rotation breaks) and the runs on the right (with no breaks). The entries with no rotation breaks have uncertainties about a factor of ten smaller than the corresponding entries with frequent breaks, but the relationships among the uncertainties on the left and on the right are similar.

VI. DISCUSSION

The scale of the results in the table is ultimately set by the measurement uncertainty in the scan direction (0.35 mas in this study). The benchmark for Table 1, then, is $log(3.5 \times 10^{-4}) = -3.46$. Thus, for all cases with spans of about 20 min, the rotation model has a larger uncertainty than a single measurement. Contrariwise, for all cases with a single span, the rotation model has an uncertainty smaller than that of a measurement. The obvious conclusion is that reducing the number of rotation corrections is an important consideration in improving the scientific output of the mission. Even if the spacecraft rotational behavior requires a more complicated model, the resulting degradation of the angular cohesion over a batch interval is much less than that introduced by frequent rotation corrections. Similarly, within the narrow limits explored here, the precession rate is a secondary issue, even if precession is so rapid as to leave gaps in the sky coverage. (Needless to say, such gaps may be very undesirable for other reasons.)

It is interesting to note that adding coefficients to the model for δ has a greater impact on the output (as measured by the ϕ cohesion) than adding coefficients for ϕ. Nonetheless, the impact of rotation breaks is greater still. In the future, a rotation model making use of nutation terms, rather than simple polynomials, might be used in a similar study, since such a model would require fewer free parameters. However, "fine-tuning" of this sort would serve little purpose while so many other aspects of the instrument's behavior remain to be investigated.

VII. REFERENCES

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VIII. FIGURE CAPTIONS

Figure 1. Geometric mean uncertainty in the difference in ϕ between pairs of points on the evenly-spaced grid, as a function of the separation (or "lag") between the points. The lags are given in hundredths of the batch interval. The plot represents an average of 16 runs, each with an independent set of Gaussian-distributed rotation spans of 20 min average length. Each run is set up identically, aside from the span lengths.

The spacecraft precession rate is 6 deg/d. Each span has a separate rotation model consisting of 3 coefficients for ϕ, 2 for α, and 1 for δ.

Figure 2. Geometric mean uncertainty in the difference in ϕ between pairs of points on the evenly-spaced grid, as a function of lag. The lags are given in hundredths of the batch interval. The plot represents a single run with no rotation breaks. The spacecraft precession rate is 6 deg/d. The overall rotation model has 3 coefficients for ϕ, 2 for α, and 1 for δ.

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