Harvard-Smithsonian Center for Astrophysics

Precision Astronomy Group

MEMORANDUM

Date: 18 June 1997 TM97-03
To: Distribution
From: R.D. Reasenberg
Subject: Effects of radiation pressure on the rotation of FAME.

I. Introduction.

Radiation pressure on the FAME spacecraft produces a force that, in general, does not pass through the center of mass. Thus, there is a radiation pressure torque. For a spacecraft that is symmetrical around the spin direction, the torque tends to increase (or decrease) the Sun-spin angle. In steady state, such a torque causes a precession of the spin vector around the Sun direction as required for an HIPPARCOS-type scan of the sky. However, a calculation based on the present spacecraft nominals and a simple model shows this torque is more than an order of magnitude bigger than that needed to make the instrument precess around the Sun direction in two months.

The FAME instrument will be sensitive to thermal input. As the spacecraft rotates, the Sun-induced thermal gradient in the instrument will rotate in the instrument-fixed coordinate system. This could give rise to a measurement bias that would depend on the instrument rotation phase, i.e., on the pointing direction. Such gradients can be significantly reduced by having a solar shield that keeps the entire spacecraft in shadow. Here we assume such a shield and investigate the radiation torque as a function of the shield geometry and center of mass location. An ideal arrangement would be for the torque to be at the level required to induce the desired (or at least an acceptable) precession rate without the need for frequent spin-correction events.

Section II summarizes the nominal parameter values and establishes the starting equations. In Section III, I introduce a simple spacecraft model and calculate the radiation force and torque. These are used in Section IV to show that, particularly for a faster spinning spacecraft, an adjustable shield would provide good control of the precession.

II. Some basic quantities and relations

Here I have gathered the physical constants and nominal spacecraft parameters used in the calculations.

Q solar radiation pressure at 1 AU on an absorbing surface
$4.54\ 10^{-6}$ kg m/s per m^2 s
from solar luminosity and AU (Allen 1976, p161)

m spacecraft mass
324 kg
from poster at PBSS meeting

I spacecraft moment of inertia
65 kg m^2
assume a uniform sphere about the size of the spacecraft shown in the MIDEX proposal

r_1 radius of nominal cylindrical spacecraft
1 m

ℓ_1 length of nominal cylindrical spacecraft
2 m

θ the angle between the spacecraft cylindrical axis and the Sun direction
45 deg
the nominal spin direction is along the spacecraft cylindrical axis

R_1 reflectivity of Sun-facing spacecraft end
0.8 used here for calculation
$R_1 < R_2$ because of the solar cells on the Sun-facing end of the spacecraft

R_2 reflectivity of shield
0.9 used here for calculation

ω_0 spin rate of the spacecraft
$8.7\ 10^{-4}$ / s
2 hours per revolution, based on the MIDEX proposal

w shield width
2 m
set by need to keep all parts of the spacecraft out of view of the Sun

In the models below, I assume that a surface is a flat plane of area A, unit normal $\hat{n}$, and specular with a reflection coefficient R. The direction to the Sun is $\hat{s}$ and the angle between $\hat{n}$ and $\hat{s}$ is φ. Then there are two components of force, normal and transverse (i.e., in plane).

$$F_n = -\hat{n}\,(1+R)\,Q\,A\cos^2(\varphi) \qquad \textbf{(1)}$$

$$F_t = t\,(1-R)\,Q\,A\cos(\varphi) \tag{2}$$

where

$$\begin{aligned} t &= \hat{n}\,(\hat{n}\cdot\hat{s}) - \hat{s} \\ |t| &= \sin(\varphi) \end{aligned} \tag{3}$$

Consider a circular plate of radius r_1 and reflectivity R_1 and with normal at $\theta = 45$ deg to the Sun direction. Neglecting the (small and model-dependent effect of) reradiation of absorbed energy, the radiation forces are: $F_n = -1.26\ 10^{-5}$ kg m / s^2; $F_t = -1.43\ 10^{-6}$ kg m / s^2. If the center of mass is 0.5 m behind and centered on the plate, the torque around it is $N = 7.1\ 10^{-7}$ kg m^2 / s^2. For an angular momentum of $J = I\omega_0 = 0.0567$ kg m^2 / s, the torque causes a precession rate of $\Omega = N/J = 1.25\ 10^{-5}$ /s = 5 deg per spacecraft rotation. This precession rate is an order faster than needed to scan the sky at the nominal rate. Much of this memorandum addresses means to make the precession rate due to radiation torque equal to the value needed for the sky scan.

III. A cylindrically symmetric model.

In this model, the spacecraft is a cylinder (r_1 by ℓ_1) with an added shield. The latter is the frustrum of a cone, with the circular spacecraft end serving as the truncation disc. The shield sweeps back from the circular (Sun facing) spacecraft end by an angle α, has an inner radius r_1, and a width w. Thus, the outer radius of the shield is $r_1 + w\cos(\alpha)$.[1] A right orthogonal coordinate system has its origin in the center of the Sun-facing circular spacecraft end. The z axis is perpendicular to and pointing away from the surface; the Sun is in the y-z plane at an angle θ from the z axis.

The radiation force on the circular end was found in Section II. Here we find the force and torque around the origin from pressure on the shield. This is done by numerical quadrature in Mathcad. With the help of a little geometry, only a 1-D integral is needed.

Let β be an azimuthal angle from the x axis toward y. For the differential area

$$dA = w\left(r_1 + \frac{w\cos(\alpha)}{2}\right)d\beta \tag{4}$$

[1] In a more realistic realization of the concept, the spacecraft is a hexagonal prism and the shield comprises six rectangular and six triangular sheets.

the center of force is at

$$\rho = \begin{pmatrix} \bar{r}\cos(\beta) \\ \bar{r}\sin(\beta) \\ (r_1 - \bar{r})\tan(\alpha) \end{pmatrix} \tag{5}$$

where

$$\bar{r} = \frac{r_1^2 + r_1 w\cos(\alpha) + w^2\cos(\alpha)^2/3}{r_1 + w\cos(\alpha)/2} \tag{6}$$

and the unit normal is

$$\hat{n} = \begin{pmatrix} \sin(\alpha)\cos(\beta) \\ \sin(\alpha)\sin(\beta) \\ \cos(\alpha) \end{pmatrix} \tag{7}$$

By combining the above equations, we obtain the force and torque on the shield

$$F_s = \int_0^{2\pi} \frac{dF}{dA}\,\frac{dA}{d\beta}\,d\beta \tag{8}$$

$$\square_s = \int_0^{2\pi} \rho \times \frac{dF}{dA}\,\frac{dA}{d\beta}\,d\beta \tag{9}$$

where dF/dA is obtained by combining Eqs 1 and 2. The total radiation force on the spacecraft is $F = F_s + F_e$, where F_e is the force on the Sun-facing spacecraft end (with components previously called F_n and F_t). The total torque on the spacecraft around a point H along the z axis is

$$\square = N_s + h\,\hat{z} \times F \tag{10}$$

where h is the distance from H to the origin (i.e., h is positive when H is inside the spacecraft --

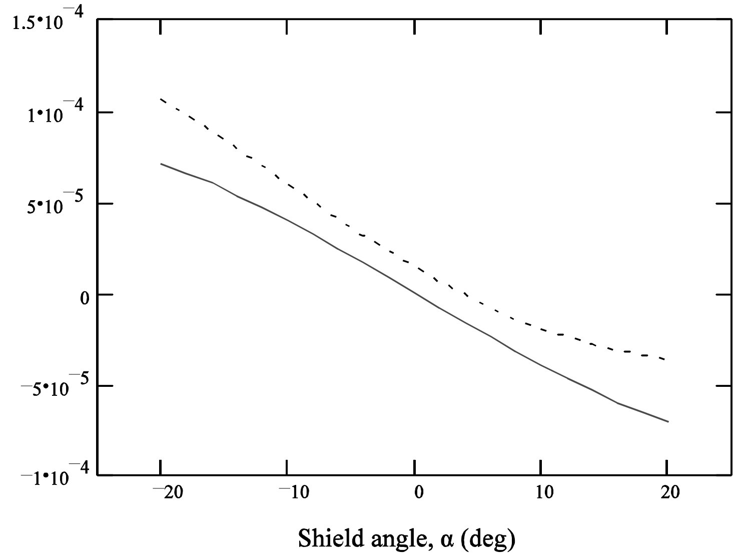

Figure 1. Torque (Nm) around two points along the cylindrical axis of the spacecraft (front, solid; 2m back, dashed) vs shield angle, deg.

behind the front end.).

I have used Mathcad to determine the torque for a variety of sets of parameters. Figure 1 shows the torque around two points on the cylindrical axis: at the front (Sun-facing) end, and behind the front end at h = 2 m. Figure 2 is the same, except the front end has been made black (R_1=0). Figure 3 shows the displacement along the cylindrical axis at which the torque is zero (for the nominal value of R_1).

IV. Precession rate and tolerances.

Initially, I assume the instrument is to precess smoothly at a rate that yields a 50% overlap in observing fields of successive rotations at the point in the rotation where the overlap is smallest.[2] Then

$$\Omega = \frac{W}{2P} \tag{11}$$

[2] This overlap requirement could likely be relaxed to near zero if the mean number of spin-correction events per instrument rotation were small. However, it is shown below that, assuming smooth precession, the precession per orbit is limited by the need to have the star images move along the CCD columns.

where Ω is the rate of change of the direction of the spacecraft cylindrical axis,[3] W is the width (on the sky) of the instrument's field of view, and $P = 2\pi/\omega$ is the instrument's rotation period. (The time required for a complete precession of the spacecraft around the Sun is $2\pi\sin(\theta)/\Omega$.) To obtain this precession rate, the required torque is

$$N_r = I\,\omega\,\Omega = \frac{\pi I W}{P^2} \qquad \textbf{(12)}$$

which, for the present nominals, yields $N_r = 5.1\ 10^{-8}$ Nm. For $\alpha = 0$, the radiation torque around a point 1 m back from the front end is $N_p = 7.13\ 10^{-6}$ Nm, and has a slope of $dN/d\alpha = -4.10\ 10^{-6}$ Nm / deg. Thus the required torque is reached at $\alpha = 1.73$ deg, where the nominal ($\alpha = 0$) torque is suppressed by two orders.

For a fixed size of focal plane array, $W \propto 1/f$, where f is the instrument focal length.[4] Further, for a fixed size ξ (in the scan direction) of a single CCD array, the single-star integration time is

$$\tau = \frac{P}{2\pi}\,\frac{\xi}{f} \qquad \textbf{(13)}$$

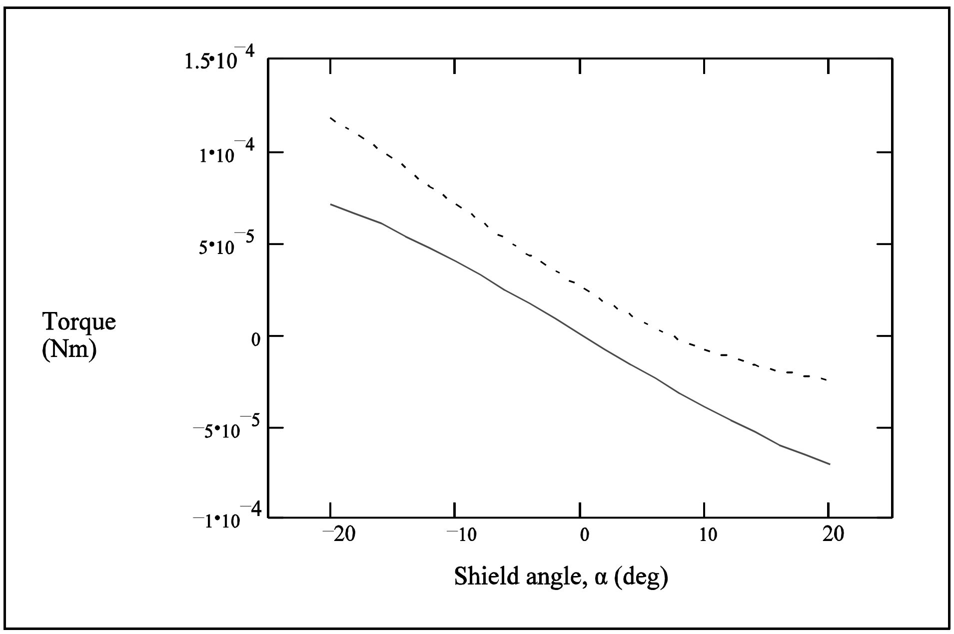

Figure 2. Like Fig. 1, but with the front surface black.

[3] Note that Ω is not the "projection" of the precession onto the z axis, which is $\Omega/\sin(\theta)$.

[4] This relation does not address the limitations that come from the design of the optics.

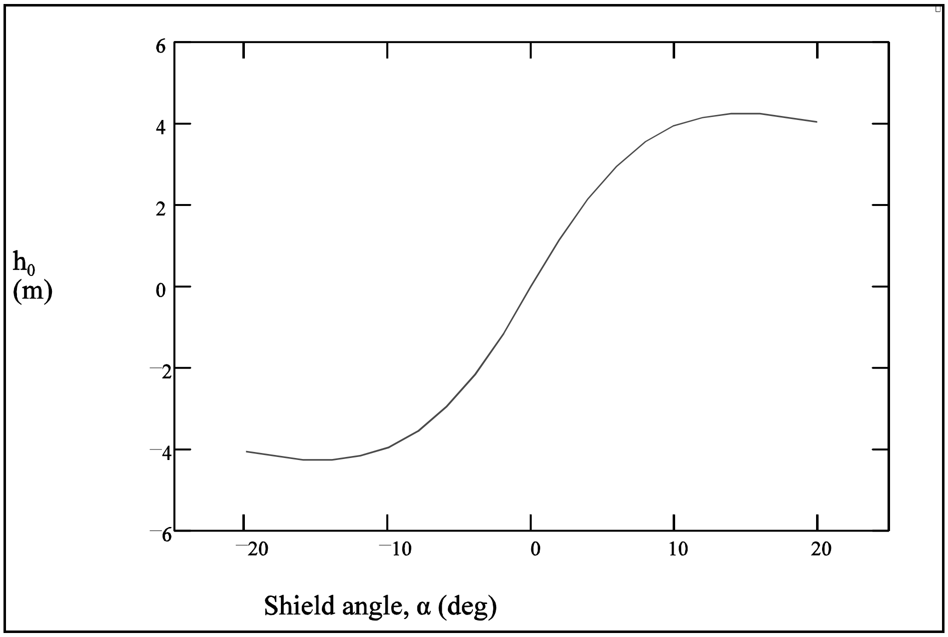

Figure 3. For the indicated shield angle (α, deg), the curve shows the distance (h_0, m) from the point along the cylindrical axis at which the torque is zero to the front end (coordinate origin).

But τ determines the limiting magnitudes (max and min), and for fixed τ, $P \propto f$. Then $N_r \propto f^{-3}$ for the case of both fixed focal plane size and fixed limiting magnitude.

Recently, Phillips has shown that the total information return continues to grow as f is decreased, with f = 6 m thought to be practical. Compared to the nominal of f = 36 m, the shorter focal length yields about four fold more information. (There are other considerations, which will be addressed in forthcoming memoranda by Phillips.) Assuming for now that the 6 m focal length is adopted, that the limiting magnitude is not shifted, and that an optical system with the corresponding large field of view is found to be viable, then there would need to be 216 times as much torque as had been anticipated. This torque could be obtained by making $\alpha \approx -1$ deg. However, as seen below, the instrument would not work well with such a high precession rate.

If the precession is to be continuous and at a constant rate, then that rate will be limited by the need to have the star image move along a column of pixels on the detector. If the precession is to produce a maximum transverse (angular) displacement D of the field of view *during one rotation* of the spacecraft, then the direction of the image path in the focal plane will vary with the instrument rotation phase by up to $\pm\epsilon$, where $\epsilon = D/2\pi$. Assume that: (a) a combination of optical anamorphism, pixel shape, and pixel co-adding yields an effective pixel aspect ratio of κ ($\approx$ 10); (b) there are 4k pixels in a column (scan direction) of a CCD; and (c) we will permit the image track to be displaced by up to δ ($\approx$ 1/2) of an effective pixel as it moves from one end to the other of the CCD. Then $D < 0.09\ \delta\kappa$ (deg); with the nominal values, $D < 0.45$ deg. This is larger than the present nominal (0.37 deg), but far smaller than might be possible with a wide field of view. (Thus, as a practical matter, we may operate in a regime in which $N_r \propto f^{-2}$.)

To address the positional tolerance of the shield, we assume that: (a) the shield is held in place by struts connecting its outer rim to a point near the back end of the spacecraft cylinder; (b) the strut lengths are fixed but that the back end of each strut can move under control along a track parallel to the cylinder axis; and (c) $\alpha = 0$ when $u = 0$, where u is the displacement of the back end of the strut toward the front of the spacecraft.[5] Then $d\alpha/du \approx -29$ deg/m and $dN/du\ -1.2\ 10^{-4}$ Nm/m. Thus, to obtain the nominal N_r (of $5.1\ 10^{-8}$ Nm, based on a spin rate of 2 hour per revolution) to 10%, requires u to be set to 0.04 mm. This is uncomfortable but possible, for example, with a 1 mm pitch screw and a stepping motor. The value of u would be adjusted using feedback (via the ground controller) from the rate of spacecraft precession. However, if the spacecraft were to be required to spin six times faster, which would required the torque to be 36 times larger, then u would need to be set to 1.5 mm. This would be quite comfortable, and the precession rate would still be 20% below the limit of $D < 0.45$ deg.

V. Discussion and effect of solar wind.

The solar wind produces a pressure about three orders smaller than the radiation and is "100% stochastic." The wind comes from a different apparent direction (low speed version of aberration) than the radiation and is not reflected ($R = 0$). It may provide a stochastic drive of the Eulerian nutation of the spacecraft. This needs to be investigated.

For the nominal spacecraft spin and precession rate, the shield must be adjusted to reduce the torque by two orders. For this purpose, the stability of the shield, shield-support, and shield-drive mechanisms needs to be investigated carefully. Further, the solar wind will provide a significant fraction of the desired torque. (The amount depends on both the intensity and direction of the wind.) Note that the solar-wind torque is not null on a shield configured to null the radiation torque.

By contrast, for a spacecraft spinning and precessing at five times the nominal rate, the required nulling is much shallower, the control correspondingly easier, and the effect of the wind correspondingly less important. In this case, there would be no need to adjust α in response to the solar wind. In the slow-spin case, such adjustment might be necessary, depending on how precisely we require the spin vector to be maintained.

---

[5] As a practical matter, the drive mechanism might be located in the front of the spacecraft where it is warm and a small extra dissipation of power would not adversely affect the performance of the instrument. The motion of the actuator could be transferred to the other end of the spacecraft by a rod. That motion could also serve to release the shield. The hinges for the shield could be made from "steel tape measure."

Thus we find that the design with a shorter focal length, which is supported by Phillips' recent analysis of information rate, makes possible a faster and "better" spacecraft rotation

VI. References.

Allen, C.W., *Astrophysical Quantities,* University of London Athlone Press, 1976.

VII. Acknowledgment

I thank M.A. Murison and J.D. Phillips for reviewing this memorandum in draft form.

VIII. Distribution

SAO:	USNO via PKS:
R.W. Babcock	M.E. Germain
J.F. Chandler	K.J. Johnston
J.D. Phillips	M.A. Murison
R.D. Reasenberg	P.K. Seidelmann
I.I. Shapiro	S.E. Urban