On the FAME Astrometric Error Distributions

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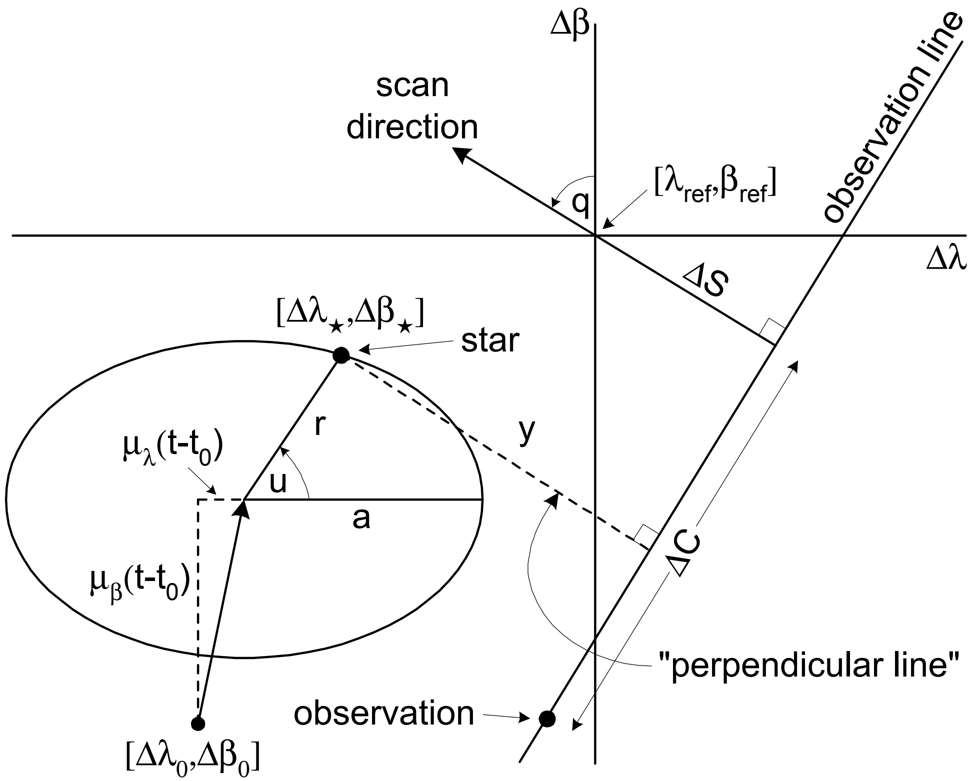
Simulation Details

- This set of simulations consisted of 4 runs
  - ψ = 36, 45, 54 deg runs with two banks of astrometric CCDs
  - One ψ = 45 deg run with four banks of astrometric CCDs
  - T_φ = 20 days and T_θ = 40 min for all cases
- Simulation length: 2.5 years
- Observations accumulated on equal-area [λ,sin β] grid
  - grid dimensions = [341,170]
  - grid cell size on ecliptic equal to FOV diameter
- Observation interval: 7.04 seconds
  - time needed to move one FOV
- Two viewports (basic angle 81.5 deg)
- Sun-tracking variation of Sun angle (~4°) NOT included
- Observation errors sampled from Gaussian error distribution
  - in-scan 1-σ error: 580 μas
- Quantities calculated at each observation:
  - scan angle q
  - ecliptic latitude & longitude (offset from cell center)

Simulation Details *(continued)*

- Least squares analysis for each grid cell
  - forthcoming memo on method
- Parameters solved for at each grid cell location:
  - position
  - proper motion
  - parallax
- Parameter errors (and correlations) from grid cell covariance matrices
- Results presented here:
  - histograms of parameter errors
  - cumulative histograms of parameter errors

Observation Geometry



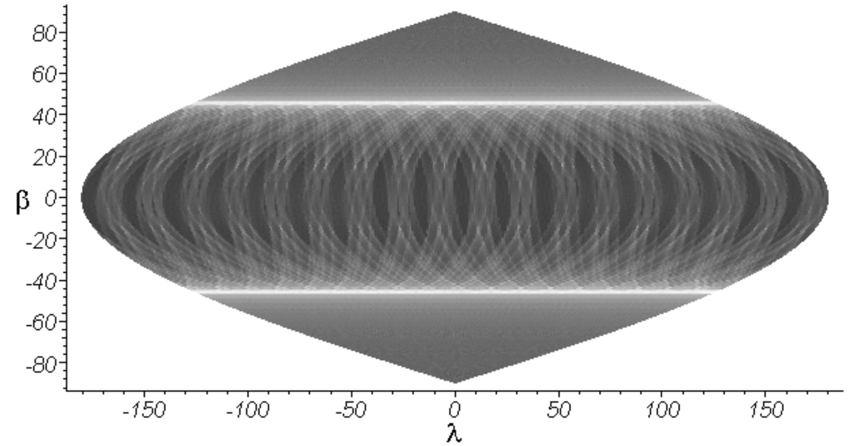
$$
\begin{aligned}
|y| \simeq \Big| \Delta S &+ \frac{a\,|\sin\beta_{ref}|\,\cos(\lambda_{ref}-\lambda_{\odot}+q)}{\sqrt{1-\cos^2\beta_{ref}\,\sin^2(\lambda_{ref}-\lambda_{\odot})}} \\
&+ [\Delta\beta_0 + (t-t_0)\mu_\beta]\cos q \\
&- [\Delta\lambda_0 + (t-t_0)\mu_\lambda]\sin q \Big|
\end{aligned}
$$

The instrument makes an observation of a star, deriving ΔS and ΔC (scan and cross-scan positions) with respect to local ecliptic coordinates [Δλ,Δβ] located on the sky at [λ_{ref},β_{ref}]. Scan direction is indicated, making an angle *q* wrt the local ecliptic meridian (Δβ axis). The observation point is not coincident with the star due to single-measurement errors. Measurement errors are in general orders of magnitude worse cross-scan than in-scan, causing the measurement error ellipse to be *extremely* elongated. We therefore approximate it as the limiting case: an "observation line". (Note that ΔC is not drawn to scale in the figure.) Given a number of observations, the distance *y* of the observation lines from the true location of the star then becomes the most natural quantity to minimize in a least squares sense.

Due to Earth's orbital motion, the star moves on an ellipse on the sky, with semimajor axis *a* and eccentricity cos β. Due to proper motion [μ_λ,μ_β], the center of the ellipse moves during the mission. The least squares algorithm minimizes the length of the perpendicular line segment *y* by solving for the astrometric parameters: (1) the position [$\Delta\lambda_0$,$\Delta\beta_0$] of the ellipse center at epoch t_0, (2) the proper motion components [μ_λ,μ_β], and (3) the semimajor axis *a* of the parallactic ellipse. The resulting covariance matrix then yields the formal errors and cross-correlations of the parameters.

General Characteristics of the Two Distributions

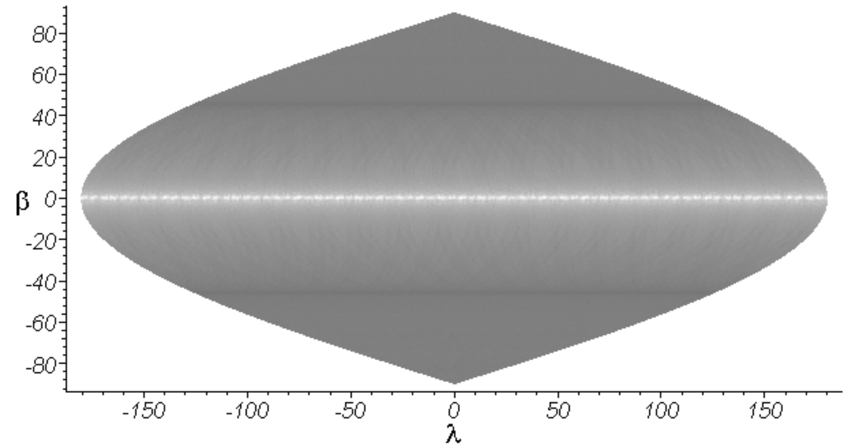
- Observation density distribution
  - highest density at top & bottom of precession cone holes (which smear in longitude), corresponding to two zones in latitude $|\beta| = 90 - \psi$
  - lowest densities are in ecliptic band between the high-density zones



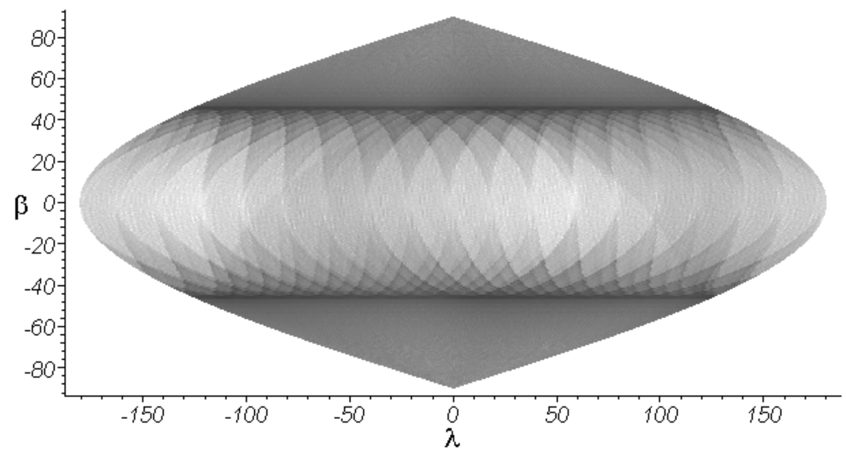
  - ecliptic band exhibits density "ribbing" corresponding to the times when the spacecraft spin axis lies in ecliptic plane
  - ➡ best accuracies should be in the mid-latitude high-density zones
  - ➡ worst accuracies should be in the ecliptic band
  - ➡ ecliptic band is not uniformly bad
- Scan angle distribution
  - homogeneous in polar cap regions (latitudes above high-density zones)
  - cone-shaped on ecliptic, with cone opening angle $90 - \psi$
  - ➡ better position accuracies in polar cap regions
  - ➡ longitude position accuracy substantially degraded near ecliptic
  - ➡ latitude position accuracy slightly degraded near ecliptic
  - ➡ better parallax accuracy in polar cap regions

All-Sky Error Distributions (45 deg. Sun Angle)

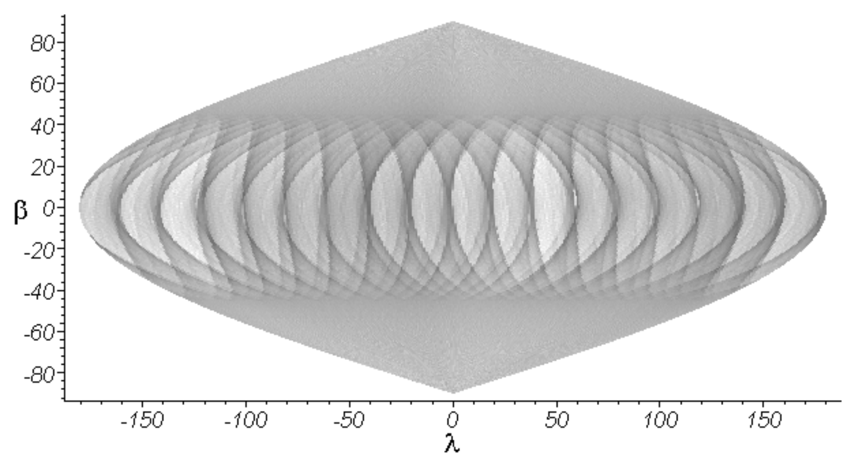
errors in parallax
(log scaling)



errors in longitude

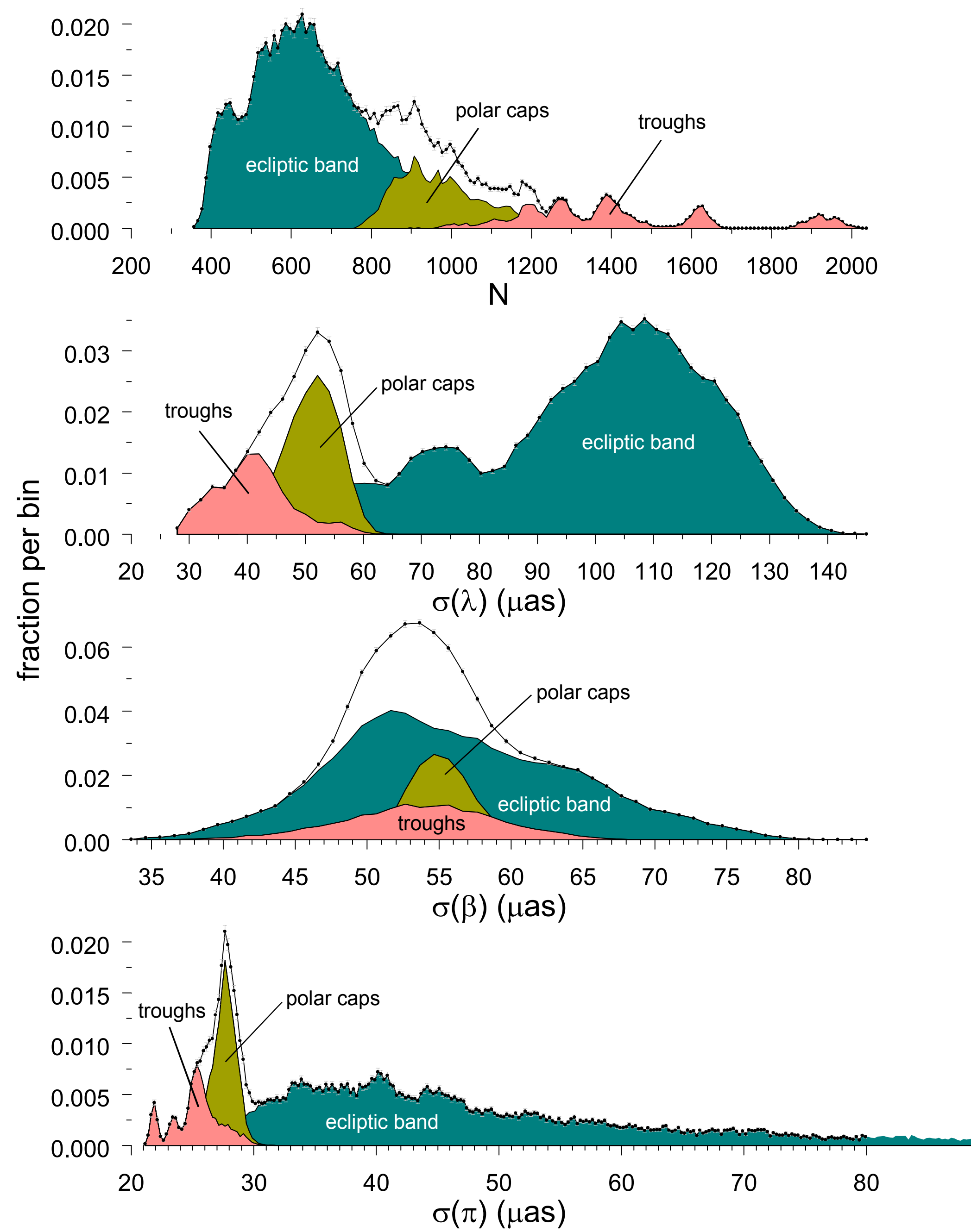


errors in latitude

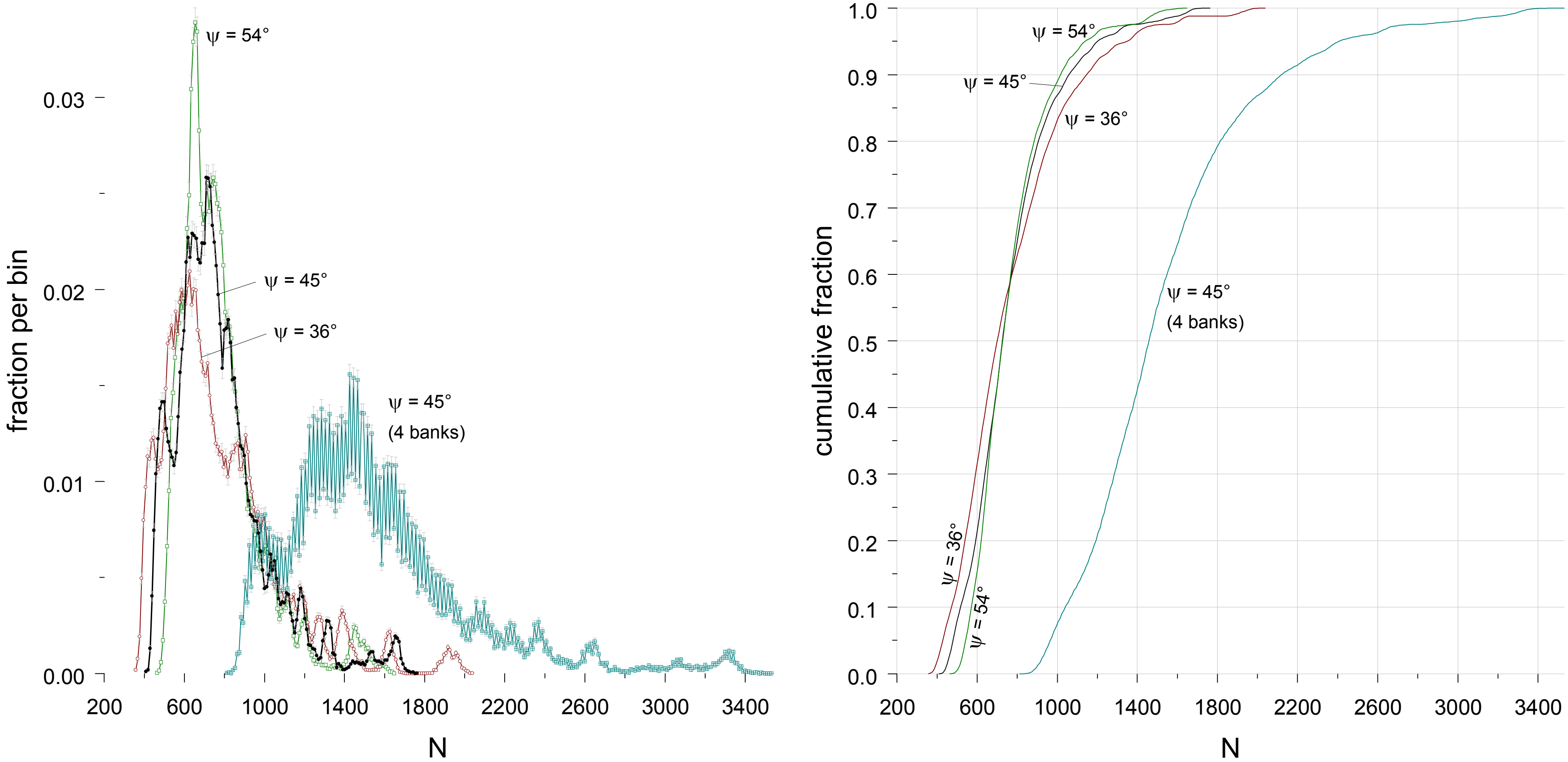


Components of Histograms and their Behavior

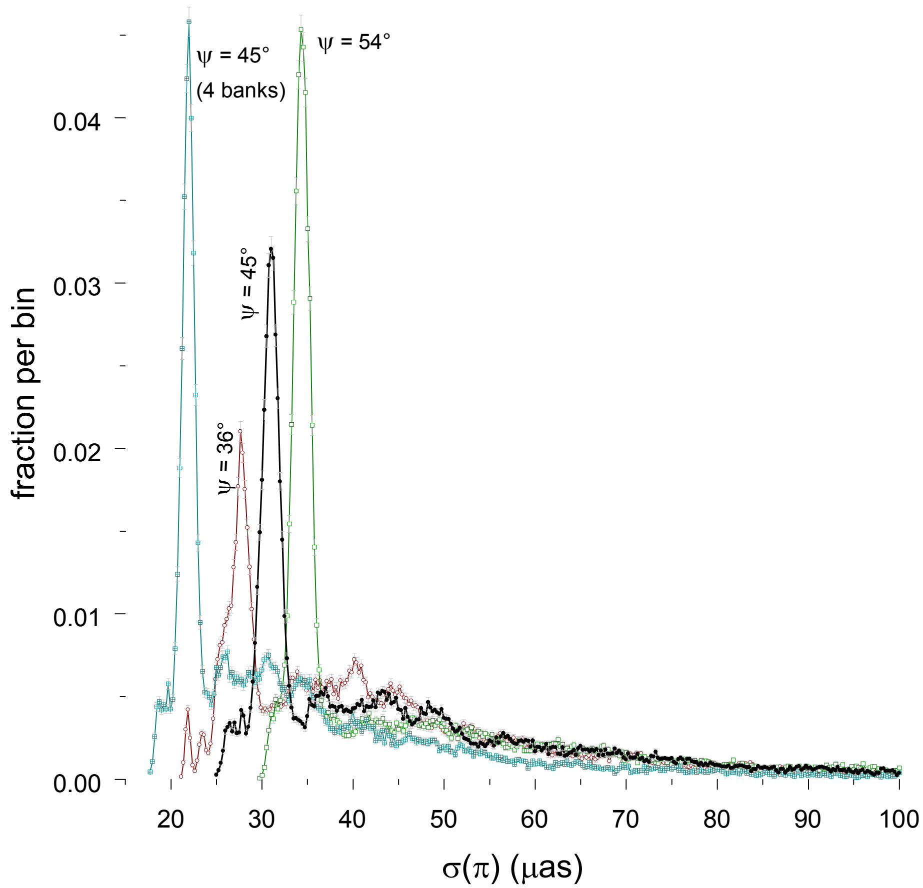
- sky naturally divided by scanning geometry into distinct regions:
  - high-density troughs at $|\beta| = 90 - \psi$
  - ecliptic band $|\beta| < 90 - \psi$
  - polar caps $|\beta| > 90 - \psi$
- as Sun angle decreases:
  - polar caps shrink
  - ecliptic band grows
  - longitude
    - high-accuracy population shrinks, moves left
    - low-accuracy population grows, moves right
  - latitude
    - distribution broadens and peak moves left
  - parallax
    - main feature shrinks, moves left
    - poor-accuracy fraction grows

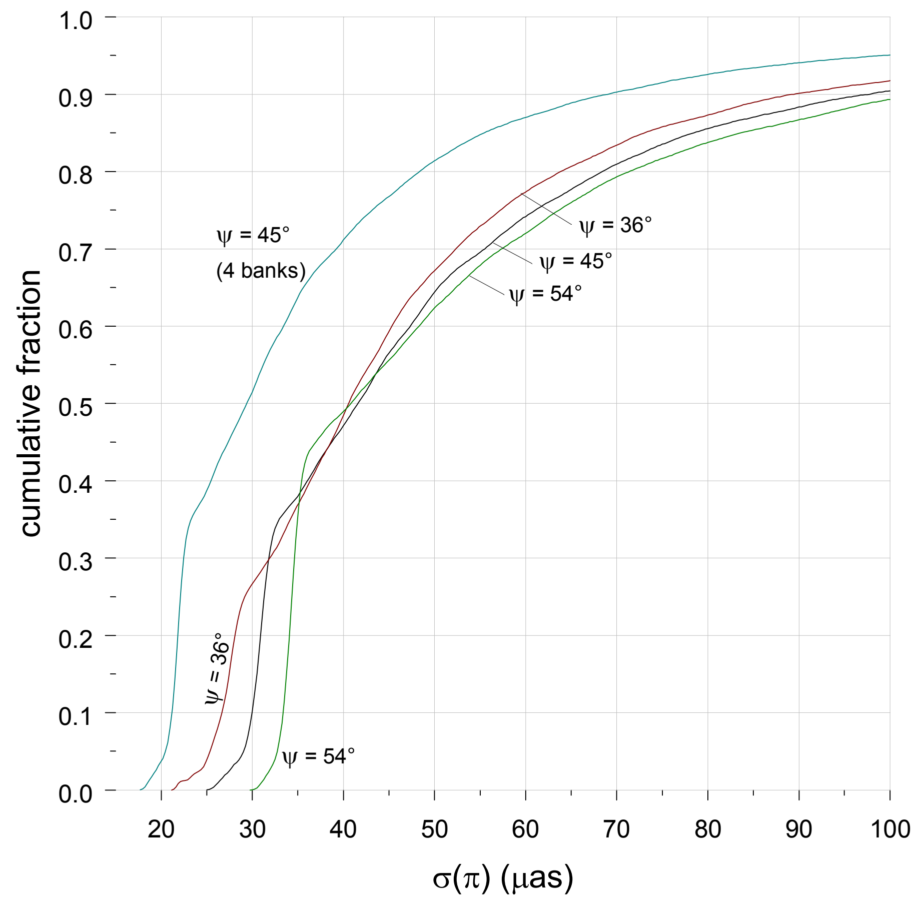


Histograms: Observation Counts



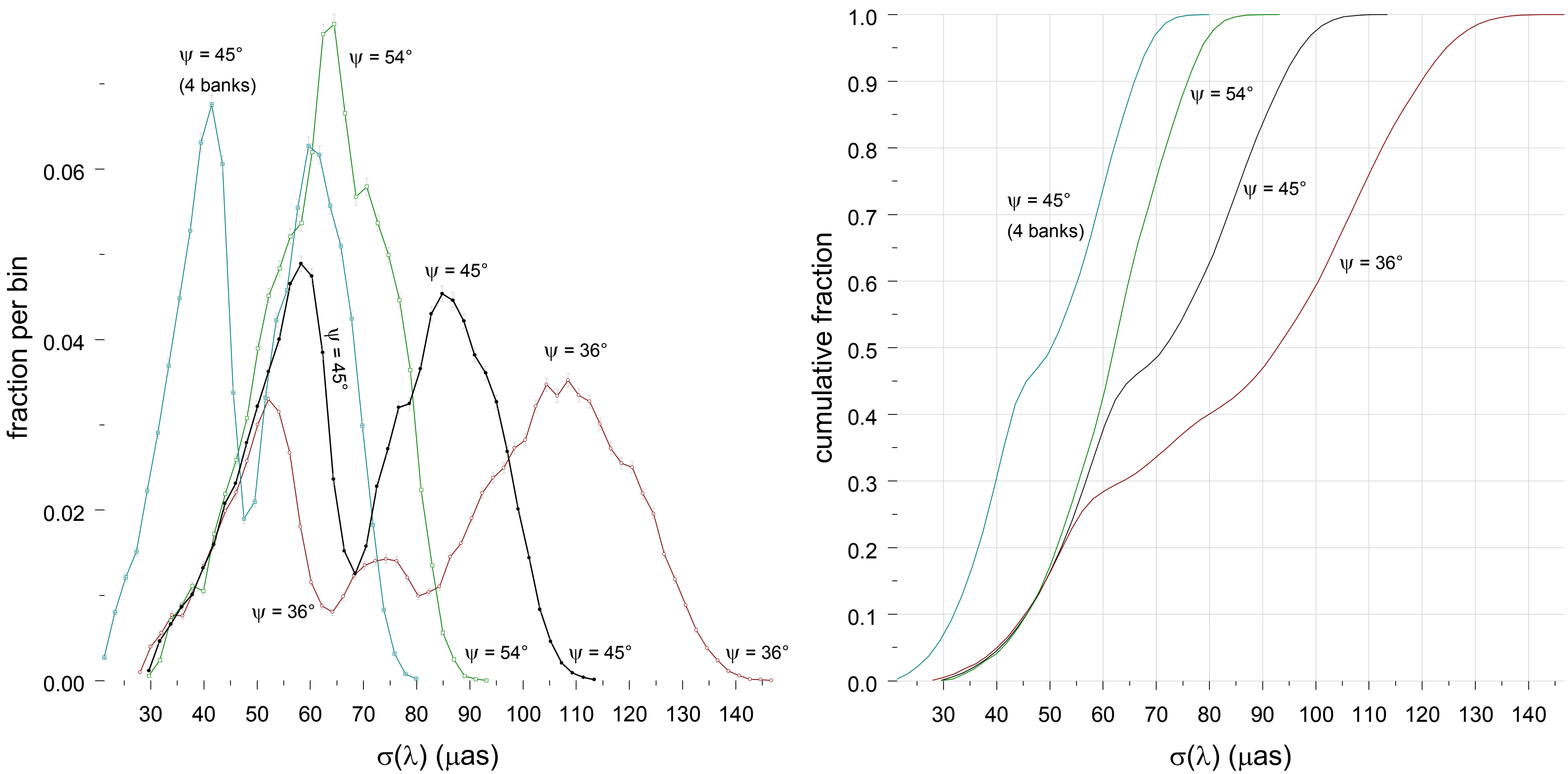
Histograms: Errors in Parallax



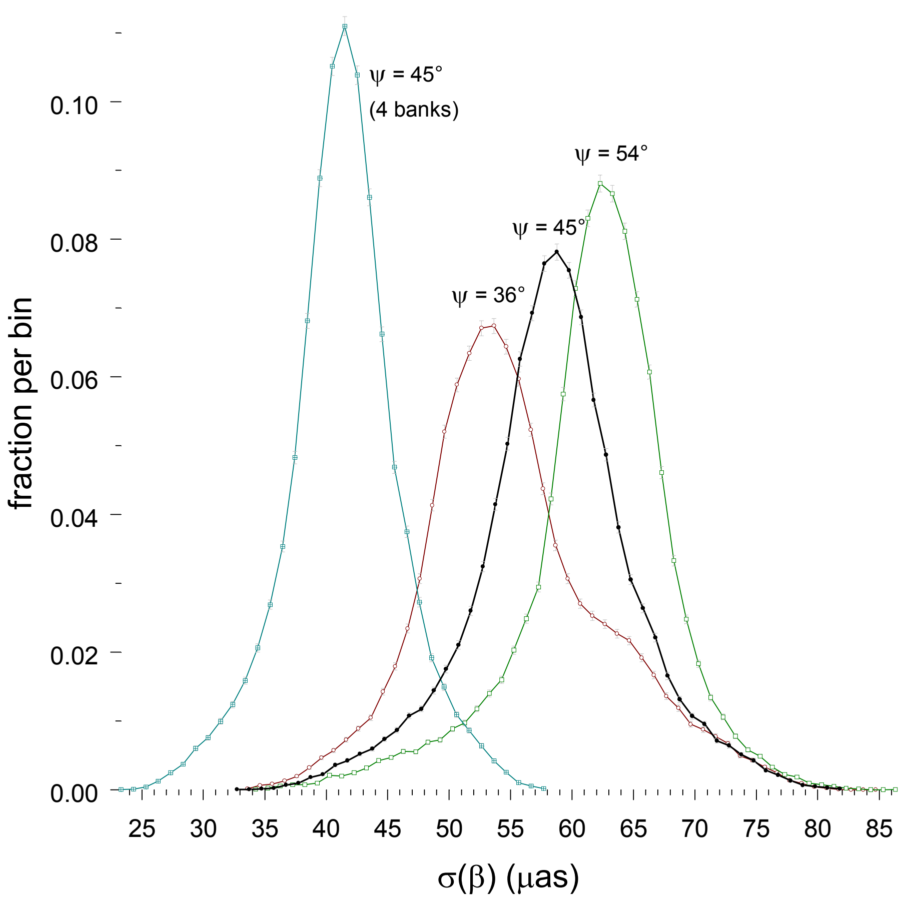


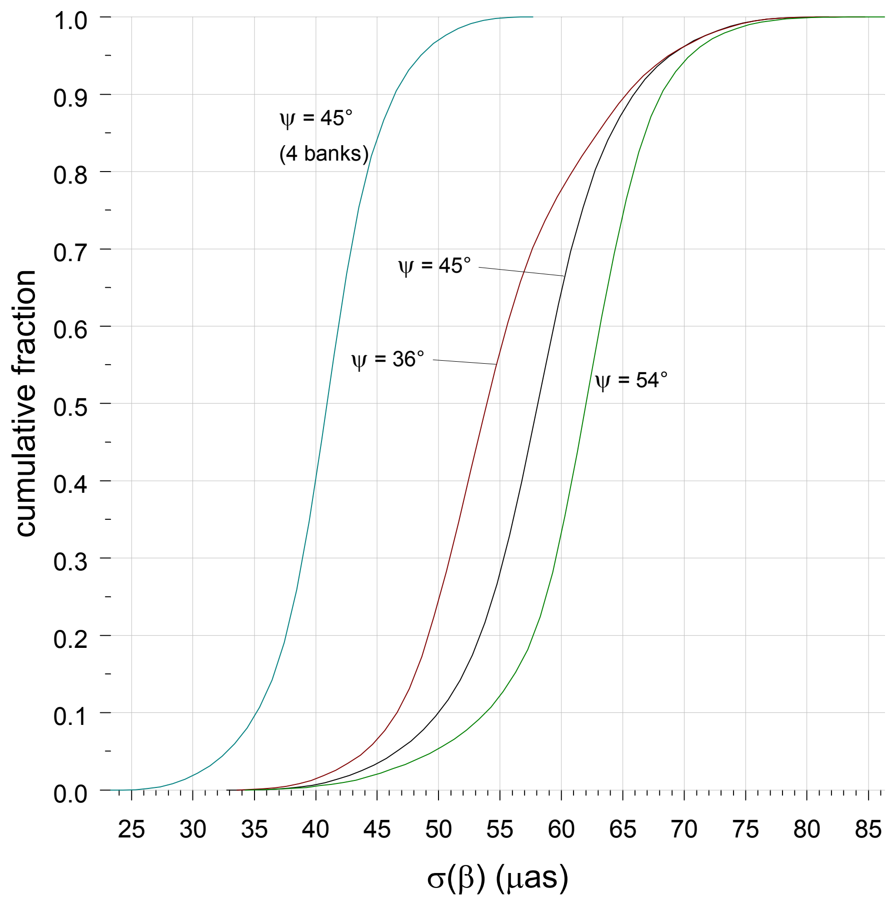
- 4-banks peak is higher due to twice the number of observations over the mission
- 2-banks cases are directly comparable

Histograms: Errors in Ecliptic Longitude

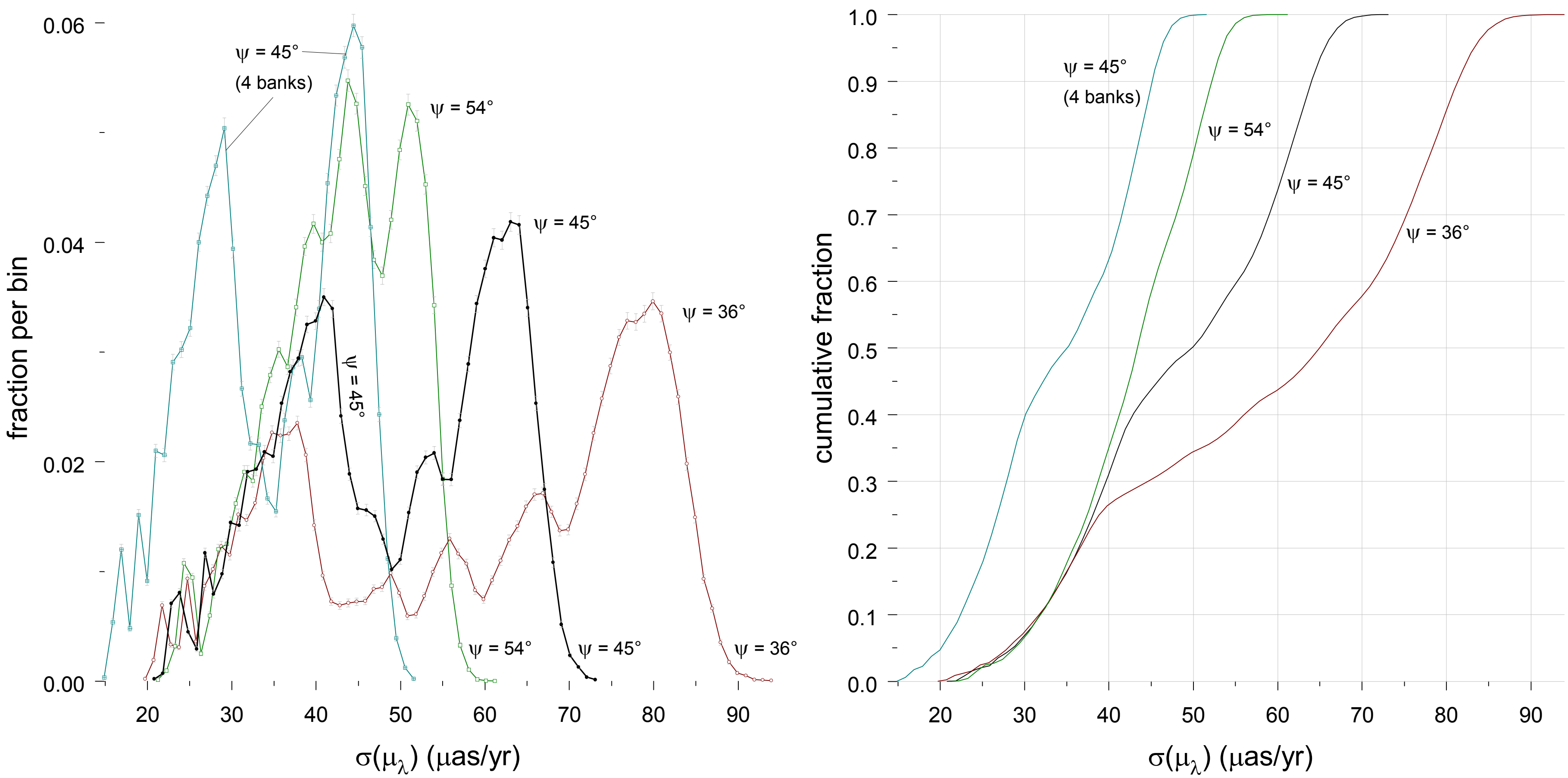


Histograms: Errors in Ecliptic Latitude

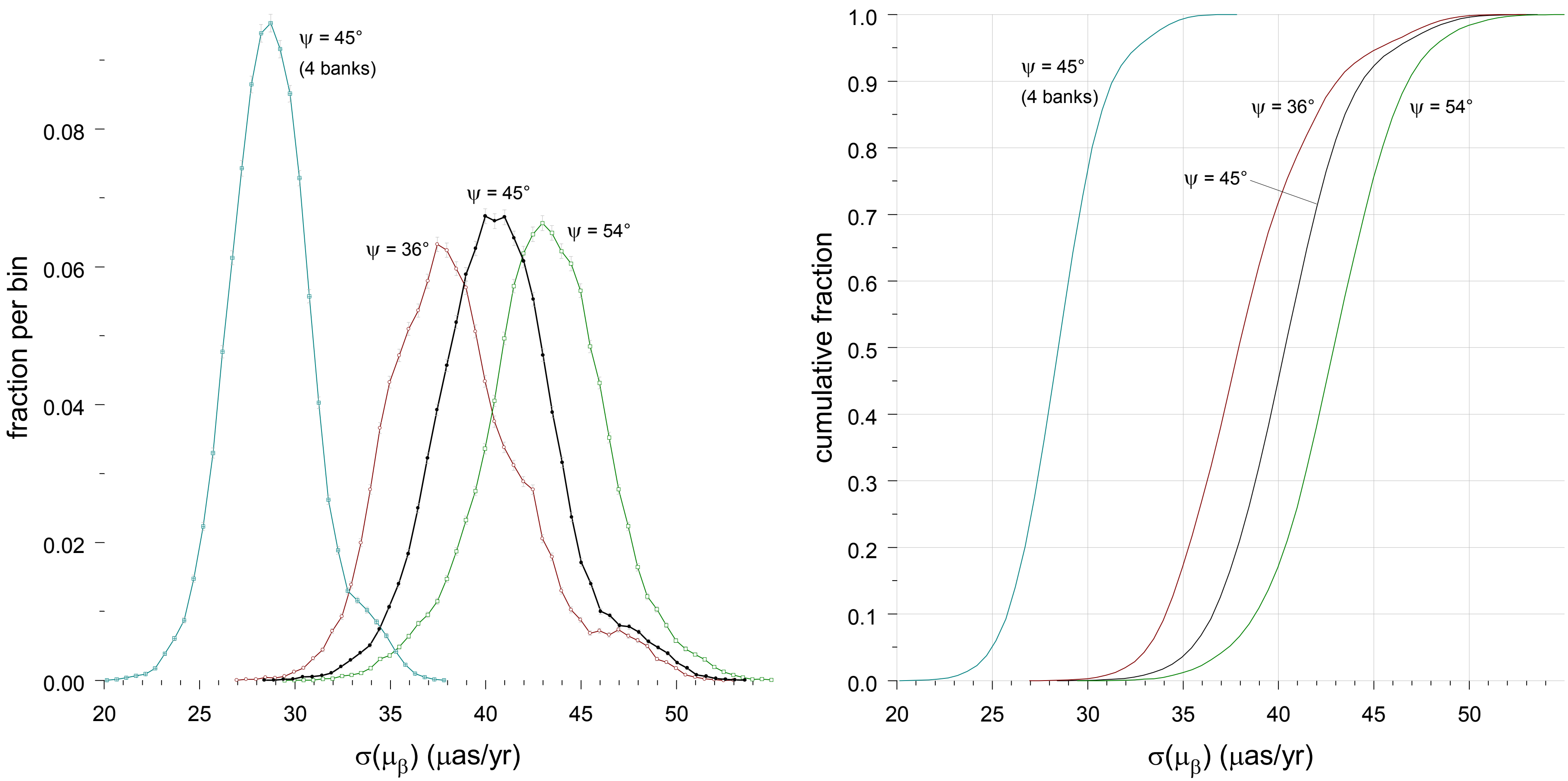




Histograms: Errors in Proper Motion in Longitude



Histograms: Errors in Proper Motion in Latitude



Comments

- All-sky images mask important quantitative details but provide useful view of distribution structure
  - A picture is worth a thousand words...but a thousand words isn't necessarily the whole story
- Histograms mask important details of distribution spatial structures but provide useful quantitative characteristics
- Parallax errors
  - distribution consists of a large, high-accuracy peak with a very long, low-accuracy tail
    - errors are terrible on and very near the ecliptic
    - high-accuracy peak is from polar cap regions
  - distribution peak is better by ~10 percent for smaller sun angle (36 vs. 45)
  - area of sky in high-accuracy peak is roughly twice as large for 54° as it is for 36°
    - [36°,45°,54°] = [25,35,45] percent
  - ➡ larger Sun angle is better
  - high-accuracy population for all cases resides below 40 μas
  - 50 μas target is met for ~65 percent (2 banks) and >80 percent (4 banks) of the sky

Comments *(continued)*

- Longitude errors
  - larger errors come from the ecliptic band region
  - smaller errors come from the high-accuracy troughs and the polar cap regions
  - high-accuracy population is more accurate but a smaller percentage of the sky for smaller Sun angle
  - low-accuracy population is less accurate and occupies a larger percentage of the sky for smaller Sun angle
  - around ~55° Sun angle the low- and high-accuracy populations merge into a single intermediate-accuracy population
  - ➡ larger Sun angle is better
  - 50 μas target is met for only a small fraction of the sky
    - 2 astrometric CCD banks: ~20 percent, independent of Sun angle
    - 4 astrometric CCD banks: ~50 percent

Comments *(continued)*

- Latitude errors
  - less range than longitude
    - degradation due to dependence of scan angle distribution on latitude is smaller than corresponding degradation of longitude errors
  - 50 μas target is met for only a small fraction of the sky with 2 CCD banks
    - ~[25,10,5] percent for [36°,45°,55°]
  - ➡ smaller Sun angle is better
  - 50 μas target is met for 97 percent of the sky with 4 CCD banks
- Proper motion errors
  - same behavior (and qualitative conclusions) as respective position errors
- Observation Counts
  - 1000-observation target is met for only a small fraction of the sky
  - ~[18,14,12] percent for [36°,45°,55°] with 2 CCD banks
  - ➡ smaller Sun angle is better
  - 1000-observation target is met for 93 percent of the sky with 4 CCD banks

Comments *(continued)*

- Generally-low accuracies argue for obtaining a higher number of independent observations
  - focal plane arrangement: 4 rows of astrometric CCDs?
  - longer mission than 2.5 years
    - additionally, would smooth out longitudinal ribbing and generally improve the astrometric quality of the distributions