ECLIPTIC COORDINATES OF THE FAME VIEWPORTS AND SYMMETRY AXIS

MARC A. MURISON
Astronomical Applications Department, U.S. Naval Observatory, 3450 Massachusetts Ave NW, Washington DC 20392
murison@aa.usno.navy.mil

June 19, 2000

1. INTRODUCTION

For various purposes, it is useful to have a description of the directions on the sky of the FAME viewports as well as the spacecraft spin axis. Given the dynamical and physical setting, the most natural coordinate system is the ecliptic coordinate system, (λ, β), where β is the ecliptic latitude and λ is the ecliptic longitude. This Note derives the ecliptic coordinates of the two viewports and the spin vector as functions of a set of Euler angles, (φ, ψ, θ), where the Euler angles connect the spacecraft body frame to an external (ecliptic) frame. These angles and their relationship to the two coordinate frames are illustrated in Figure 1 below.

Viewport 1 is the leading viewport, viewport 2 is trailing. The body frame coordinate system is $[x, y, z]$. The y axis coincides with the viewport 1 direction. Viewport 2 lies in the xy plane and trails viewport 1 by an angle γ (the so-called *basic angle*). The z axis is along the spacecraft symmetry axis (i.e., the spin axis). The external frame $[X, Y, Z]$ is aligned with the Sun such that the Z axis points towards the Sun and the XZ plane lies in the plane of the ecliptic. The Y axis points towards the north ecliptic pole. θ is the fast Euler angle, ψ is the Sun angle, and φ is the precession angle.

2. ECLIPTIC COORDINATES OF VIEWPORT 1

In Figure 2, (λ, β) are ecliptic longitude and latitude of the two viewports, and $\lambda_\odot$ is the ecliptic longitude of the Sun. (The Z axis points towards the Sun.) In this figure the spacecraft spin vector points below the XZ plane. From the large spherical triangle composed of smaller triangles A and B, we have

$$\sin\beta_1 = -\sin\varphi\sin\theta + \cos\varphi\cos\theta\cos\psi \tag{1}$$

$$-\sin\theta = \sin\beta_1\sin\varphi + \cos\beta_1\cos\varphi\sin(\lambda_1 - \lambda_\odot) \tag{2}$$

$$\cos\theta\sin\psi = \cos\beta_1\cos(\lambda_1 - \lambda_\odot) \tag{3}$$

Putting eq. (1) into eq. (2), we find

$$-\sin\theta\cos\varphi = \cos\psi\cos\theta\sin\varphi + \sin(\lambda_1 - \lambda_\odot)\cos\beta_1 \tag{4}$$

Now, $\cos\beta_1 \geq 0$ for all β_1. We may therefore make explicit use of (1) and set $\cos\beta_1 = \sqrt{1-\sin^2\beta_1}$ in (3) and (4), obtaining expressions for λ_1.

$$\cos(\lambda_1 - \lambda_\odot) = \frac{\cos\theta\sin\psi}{\sqrt{1-(\cos\varphi\cos\theta\cos\psi - \sin\varphi\sin\theta)^2}} \tag{5}$$

$$\sin(\lambda_1 - \lambda_\odot) = -\frac{\cos\theta\sin\varphi\cos\psi + \sin\theta\cos\varphi}{\sqrt{1-(\cos\varphi\cos\theta\cos\psi - \sin\varphi\sin\theta)^2}} \tag{6}$$

Expanding $\sin(\lambda_1 - \lambda_\odot)$ and $\cos(\lambda_1 - \lambda_\odot)$ in (5) and (6), we arrive at a prescription (along with eq. (1)) for determining the ecliptic coordinates of viewport 1 as a function of the Euler angles and of the longitude of the Sun:

$$\cos\lambda_1 = \frac{\cos\theta\sin\psi\cos\lambda_\odot + (\sin\theta\cos\varphi + \cos\theta\sin\varphi\cos\psi)\sin\lambda_\odot}{\sqrt{1-(\cos\varphi\cos\theta\cos\psi - \sin\varphi\sin\theta)^2}} \tag{7}$$

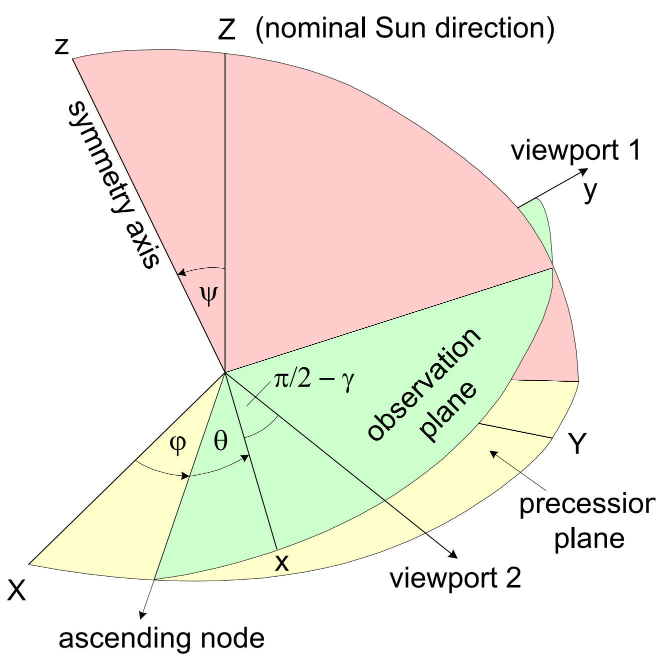

Figure 1 — Spacecraft body frame $[x, y, z]$, external frame $[X, Y, Z]$, and the Euler angles that connect them.

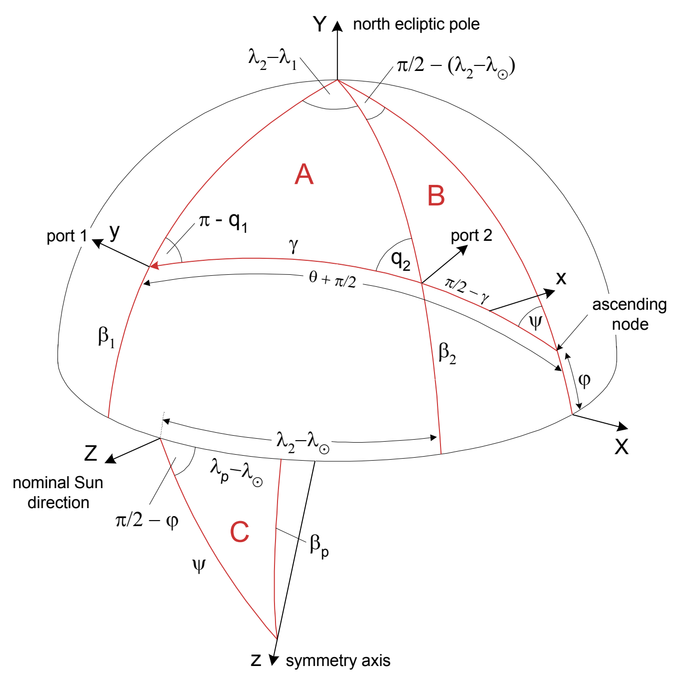

Figure 2 — Spherical geometry of the problem.

$$\sin\lambda_1 = \frac{\cos\theta\sin\psi\sin\lambda_\odot - (\sin\theta\cos\varphi + \cos\theta\sin\varphi\cos\psi)\cos\lambda_\odot}{\sqrt{1-(\cos\varphi\cos\theta\cos\psi - \sin\varphi\sin\theta)^2}} \tag{8}$$

One can view this as a coordinate transformation, given values of Sun angle ψ and solar longitude $\lambda_\odot$, from the Euler coordinates (φ,θ) to the ecliptic coordinates $(\lambda,\beta)$of viewport 1. The transformation is undefined at the points where the denominators of (7) and (8) are zero. Setting

$$\sqrt{1-(\cos\varphi\cos\theta\cos\psi - \sin\varphi\sin\theta)^2} = 0$$

we find the following eight solutions:

$$\theta = \{0,\pi\}, \quad \psi = \cos^{-1}\left(\frac{1}{\cos\varphi}\right)$$
$$\theta = \{0,\pi\}, \quad \psi = \pi - \cos^{-1}\left(\frac{1}{\cos\varphi}\right)$$
$$\psi = \cos^{-1}\left(\frac{\pm 1 \pm |\sin\varphi||\sin\theta|}{\cos\theta\cos\varphi}\right)$$

These solutions are satisfied for real values of the parameters at only the following eight specific points:

$$[\theta = \{0,\pi\}, \psi = 0, \varphi = 0] \quad [\theta = \{0,\pi\}, \psi = \pi, \varphi = \pi]$$
$$[\theta = \{0,\pi\}, \psi = \pi, \varphi = 0] \quad [\theta = \{0,\pi\}, \psi = 0, \varphi = \pi]$$

The corresponding values of the ecliptic latitude β_1 are

$$\beta_1 = \pm\frac{\pi}{2}$$

So the trouble spots are at the ecliptic poles, as one might expect. In Figure 3, the red curves are curves of constant φ (evenly spaced), as θ varies in the range $[0, 2\pi]$. The circular blue curves are curves of constant θ (again, for evenly-spaced values), as φ varies in the range $\left[0,\frac{\pi}{2}\right]$. The abscissa $\Delta\lambda = \lambda - \lambda_\odot$, and the Sun angle was set to $\psi = \frac{\pi}{4}$. The zones of avoidance show in a graphic way the need for observations throughout the year, so that the holes slide along in longitude and, over a long enough time, the sky coverage is somewhat evened out.

3. ECLIPTIC COORDINATES OF VIEWPORT 2

From Figure 2, it is apparent that the ecliptic coordinates of viewport 2 can be obtained from eqs. (1), (7), and (8) by replacing θ with $\theta-\gamma$. Hence,

$$\sin\beta_2 = -\sin\varphi\sin(\theta-\gamma) + \cos\varphi\cos(\theta-\gamma)\cos\psi \tag{9}$$

$$\cos\lambda_2 = \frac{\cos(\theta-\gamma)\sin\psi\cos\lambda_\odot + [\sin(\theta-\gamma)\cos\varphi + \cos(\theta-\gamma)\sin\varphi\cos\psi]\sin\lambda_\odot}{\sqrt{1-[\cos\varphi\cos(\theta-\gamma)\cos\psi - \sin\varphi\sin(\theta-\gamma)]^2}} \tag{10}$$

$$\sin\lambda_2 = \frac{\cos(\theta-\gamma)\sin\psi\sin\lambda_\odot - [\sin(\theta-\gamma)\cos\varphi + \cos(\theta-\gamma)\sin\varphi\cos\psi]\cos\lambda_\odot}{\sqrt{1-[\cos\varphi\cos(\theta-\gamma)\cos\psi - \sin\varphi\sin(\theta-\gamma)]^2}} \tag{11}$$

4. ECLIPTIC COORDINATES OF THE SPACECRAFT SYMMETRY AXIS

From spherical triangle C in Figure 2, we have

$$\sin\beta_p = -\sin\psi\cos\varphi \tag{12}$$

$$\cos\psi = \cos\beta_p\cos(\lambda_p - \lambda_\odot) \tag{13}$$

$$\cos\beta_p = \cos\psi\cos(\lambda_p-\lambda_\odot) + \sin\psi\sin\varphi\sin(\lambda_p-\lambda_\odot) \tag{14}$$

Substitute (13) into (14) to get

$$\cos\beta_p\sin(\lambda_p-\lambda_\odot) = \sin\psi\sin\varphi \tag{15}$$

Now, $\cos\beta_p \geq 0$ for all β_p. Hence, set $\cos\beta_p = \sqrt{1-\sin^2\beta_p}$ in (13) and (15) to obtain expressions for λ_p.

$$\cos(\lambda_p-\lambda_\odot) = \frac{\cos\psi}{\sqrt{1-\sin^2\psi\cos^2\varphi}} \tag{16}$$

$$\sin(\lambda_p-\lambda_\odot) = \frac{\sin\psi\sin\varphi}{\sqrt{1-\sin^2\psi\cos^2\varphi}} \tag{17}$$

Solve (16) and (17) for λ_p and we have a prescription (along with eq. (12)) for determining the ecliptic coordinates of the spacecraft spin (symmetry) axis as a function of the Euler angles and the longitude of the Sun:

$$\cos\lambda_p = \frac{\cos\psi\cos\lambda_\odot - \sin\psi\sin\varphi\sin\lambda_\odot}{\sqrt{1-\sin^2\psi\cos^2\varphi}} \tag{18}$$

$$\sin\lambda_p = \frac{\cos\psi\sin\lambda_\odot + \sin\psi\sin\varphi\cos\lambda_\odot}{\sqrt{1-\sin^2\psi\cos^2\varphi}} \tag{19}$$

One can view this also as a coordinate transformation, from the Euler coordinates (φ,ψ) to the ecliptic coordinates (λ,β) of the spin vector. In Figure 4, the red, closed curves are curves of constant ψ, as φ varies in the range $[0, 2\pi]$. The blue, open curves are curves of constant φ, as ψ varies in the range $\left[0,\frac{\pi}{2}\right]$. The abscissa $\Delta\lambda = \lambda - \lambda_\odot$.

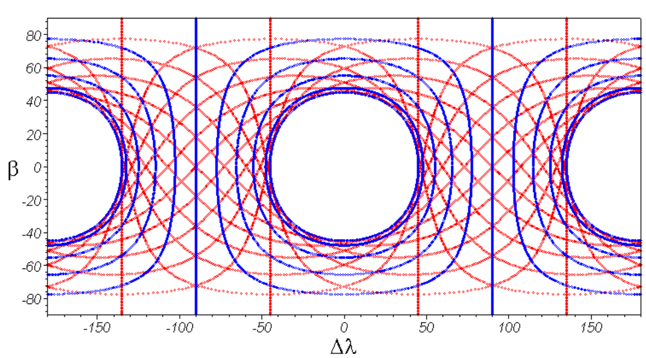

Figure 3 — Ecliptic coordinates of viewport 1.

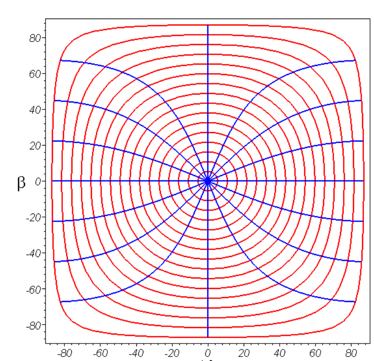

Figure 4 — Ecliptic coordinates of the spacecraft pole.