Harvard-Smithsonian Center for Astrophysics

Precision Astronomy Group

To:	Distribution	14 November 2000 TM00-03
From:	J.D. Phillips	
Subject:	Blur and offset due to image motion	

In FAME, spurious in-scan motion both blurs and offsets the stellar images. This memo calculates these effects for a sinusoidal perturbation of any frequency, as an aid in setting requirements for stability of both the in-scan rotation and the basic angle. I neglect here cross-scan motion and other effects such as flux escaping the extraction window. These will need to be considered separately.

Consider a perturbation to the in-scan look direction

$$\theta(t) = \sqrt{2}\,\sin(\omega t + \varphi) \tag{1}$$

where ω is the angular frequency of the perturbation and φ is the phase of the start of the stellar integration (t=0) with respect to the perturbation. The amplitude has been chosen so that the perturbation has a unit RMS over one perturbation cycle. The perturbation may arise from spacecraft attitude variations, from instrument structure vibration and drift, from changes in the basic angle, or from variations in the onboard clock. Blur and offset both result, in some measure, from perturbations at any frequency.

Blur. The blur due to irregular look direction motion is to be convolved with other blurring effects, and with the PSF, which is of the form $\mathrm{sinc}^2(x)$ (for the current aperture, in which there is no central obscuration [Phase B Kickoff Meeting, Sept. 6-7, 2000]). The convolution of two Gaussian blurring functions is a Gaussian whose width is the RSS of the given functions' widths. This RSS rule is approximately true for many ordinary distributions, for example, when convolving a "square wave" blur, one uniformly distributed over a range of angles, with the PSF [Phillips 2000]. Therefore, as a measure of the blur due to image motion, I take the RMS of the perturbation (1) about its mean over the stellar integration.

The mean of (1) is

$$\begin{aligned} \bar{\theta} &= \frac{1}{\tau}\int_0^{\tau} \theta(t)\,dt \\ &= \frac{\sqrt{2}}{\omega\tau}\left[\cos(\varphi) - \cos(\omega\tau + \varphi)\right] \end{aligned} \tag{2}$$

where τ is the integration duration (nominally 1.56 sec). The mean square of (1) is

$$\overline{\theta^2} = \frac{1}{\tau}\int_0^\tau [\theta(t)]^2\, dt = \frac{1}{\omega\tau}\left[\omega\tau + \sin(\varphi)\cos(\varphi) - \sin(\omega\tau+\varphi)\cos(\omega\tau+\varphi)\right] \qquad \textbf{(3)}$$

(Simpler forms of (2) and (3), but ones that are less useful later, are obtained by integrating from $-\tau/2$ to $\tau/2$.) The mean square blur over one integration is $\overline{\theta^2} - \overline{\theta}^2$. The RMS of $\overline{\theta^2} - \overline{\theta}^2$ over phase is

$$\theta_b = \left[\frac{1}{2\pi}\int_0^{2\pi}\left(\overline{\theta^2} - \overline{\theta}^2\right)d\varphi\right]^{1/2} = \sqrt{1 - \left[\operatorname{sinc}\left(\frac{\omega\tau}{2}\right)\right]^2} \qquad \textbf{(4)}$$

where

$$\operatorname{sinc}(x) = \frac{\sin x}{x}\ . \qquad \textbf{(5)}$$

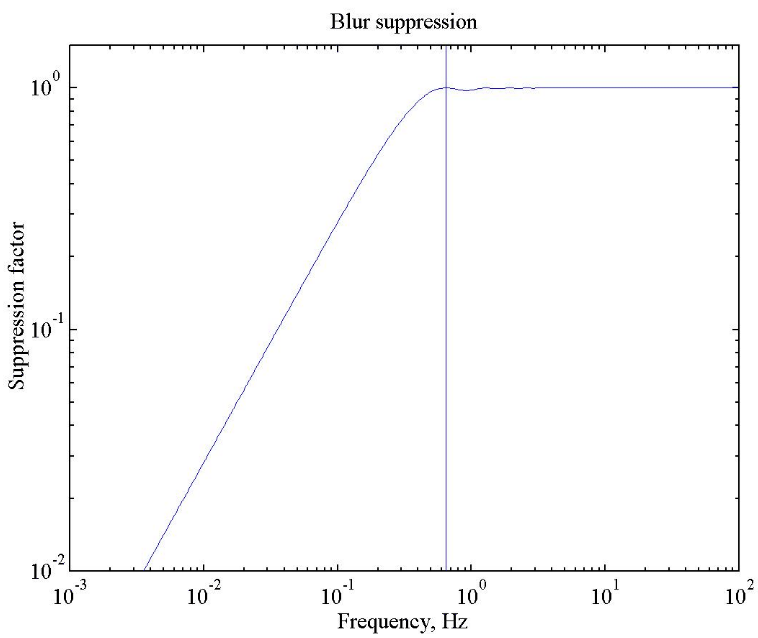

Figure 1. Blur suppression vs. frequency. The vertical line is at one cycle per integration time.

Since the perturbation was taken to have a unit RMS, (4) is the factor by which the perturbation is suppressed. It is plotted in Fig.1. As expected, for $\omega \gg 1/\tau$, there is little suppression, and for $\omega \ll 1/\tau$ there is substantial suppression.

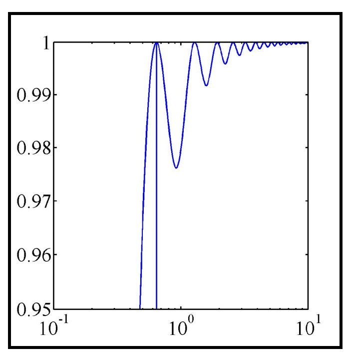

Figure 2. Enlargement of Figure 1 near one cycle per integration time.

Offset -- averaging during stellar integration. I approximate the offset as the average of the perturbation over the stellar integration.

An estimate of the expected offset due to several uncorrelated contributions is their RSS. As with blur, then, I take the RMS over phase, φ.

The mean offset for one observation is the mean of (1), which is (2). The RMS offset is

$$\theta_{\mathrm{off}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} [\theta(\varphi)]^2 \, d\varphi} = \left| \operatorname{sinc}\left(\frac{\omega\tau}{2}\right) \right| . \tag{6}$$

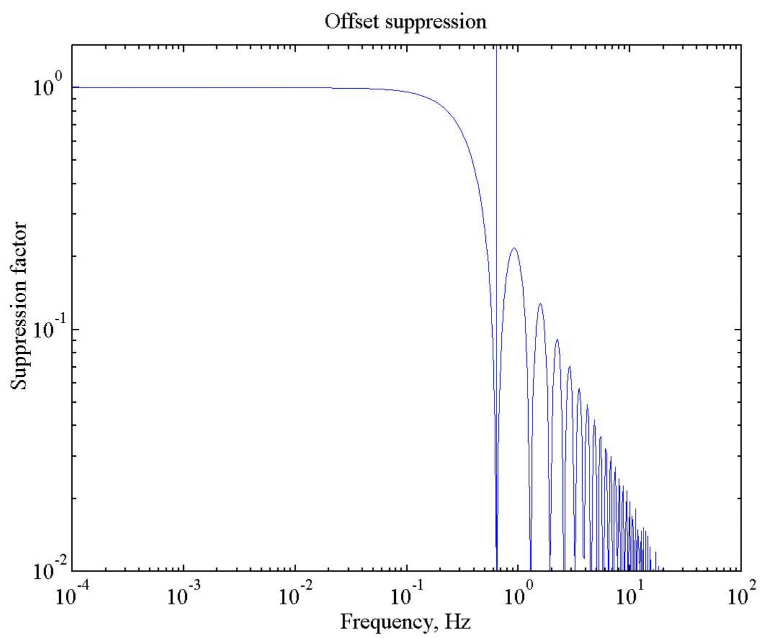

Figure 3. Offset suppression vs. frequency. The vertical line is at one cycle per integration time.

This is again a suppression factor, and is plotted in Fig.3. Note that $\theta_b^2 + \theta_{\mathrm{off}}^2 = 1$, which implies that a disturbance at any frequency is manifest as either blur or offset.

Offset -- removal *via* data reduction. The above shows a suppression of offset from high frequency perturbations, due to averaging over the observation. Low frequency perturbations to spacecraft attitude and basic angle will be substantial, and must be removed in the data reduction, at first, in the spiral stage[1]. The extent of this suppression is impossible to predict reliably without a simulation of the spiral stage, which will not be available in the near future. We may get an idea of the suppression by assuming that in one rotation, in which ~3600 stars are observed, some fraction of this number of parameters may be estimated. Many of these will be parameters of the spacecraft rotation model.

[1] The modelling of spacecraft attitude and basic angle variations can likely be improved by revisiting the spiral stage after the global reduction and determination of mission-average positions of grid stars. The mission-average grid star positions will be more accurate than the positions determined from one spiral alone. Also, in the second and subsequent spiral iterations of the spiral stage, the star positions need not be solved for (or at least could be given strong *a priori* estimates).

The actual analysis may well integrate the rigid body equations of motion for the spacecraft, using the adjustable parameters to describe the input solar torque. The resulting model will incorporate what is learned before and after launch about the spacecraft and perturbing torques. Lacking that knowledge at present, and lacking such a treatment, I consider sinusoidal variations of attitude at a wide range of frequencies, and assume that the data analysis removes completely a Fourier time series up to some frequency. These are most efficiently computed with an FFT. With this series having sine and cosine terms with from one to 60 cycles per rotation, the suppression factors in Figure 4 are obtained.

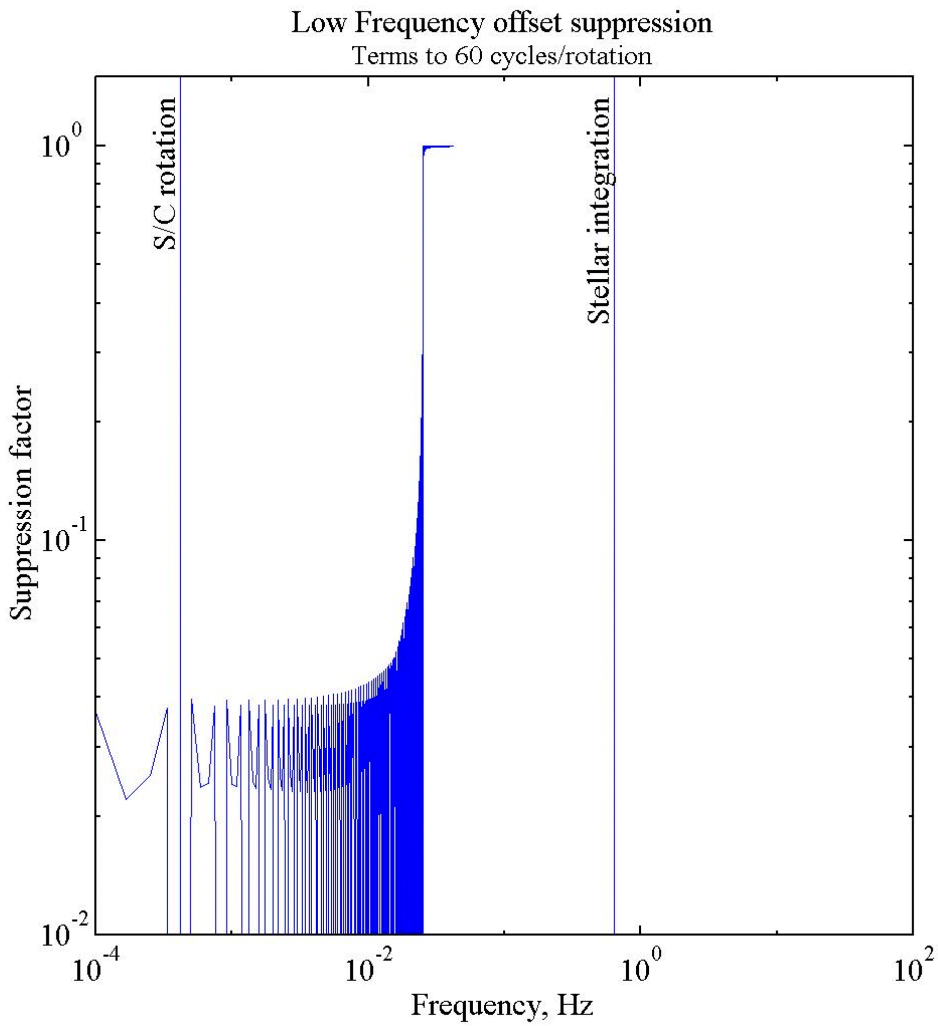

Figure 4. Removal of offset in data reduction. Vertical lines are at one cycle per rotation, and one cycle per integration.

Abbreviations

FFT	Fast Fourier transform
PSF	Point spread function
RMS	Root mean square
RSS	Root sum squared

References.

Phillips, J.D., "Blurring of FAME observations by low-frequency image motion", SAO Precision Astronomy Group Technical Memorandum TM00-xx, 2000, in preparation.