Harvard-Smithsonian Center for Astrophysics

Precision Astronomy Group

To: Distribution — 20 November 2000 — TM00-04
From: J.D. Phillips
Subject: Transverse tilts of the FAME compound mirror

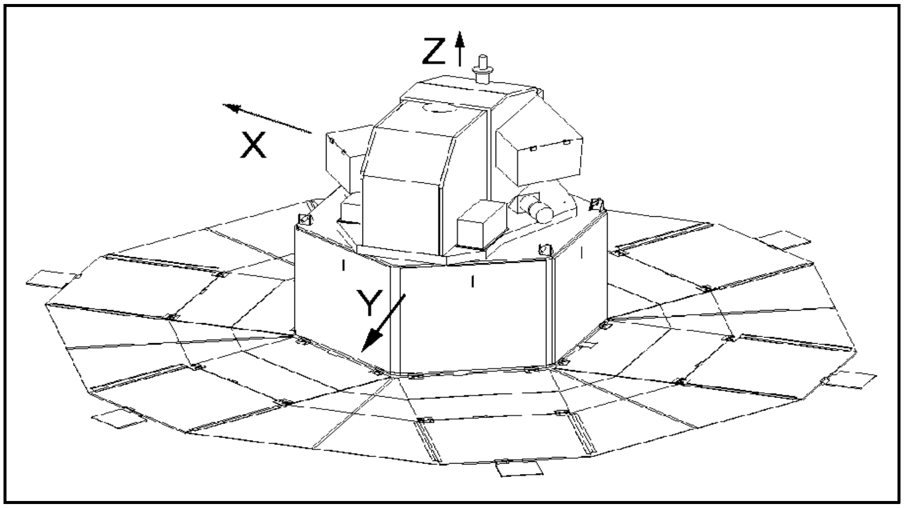

Figure 1. Instrument coordinates [FAME Concept Study Report, p. 4-72].

The FAME basic angle γ (nominally 81.5°) is established by the compound mirror, a pair of plane mirrors mounted very stably at a relative angle of γ/2. Their normals lie nominally in the instrument xy plane (see Fig. 1). This memo addresses the effect of a relative "out of plane" tilt (defined below). Rotation of the compound mirror as a whole affects the alignment of the focal plane to the rotation axis, and will need to be treated elsewhere.

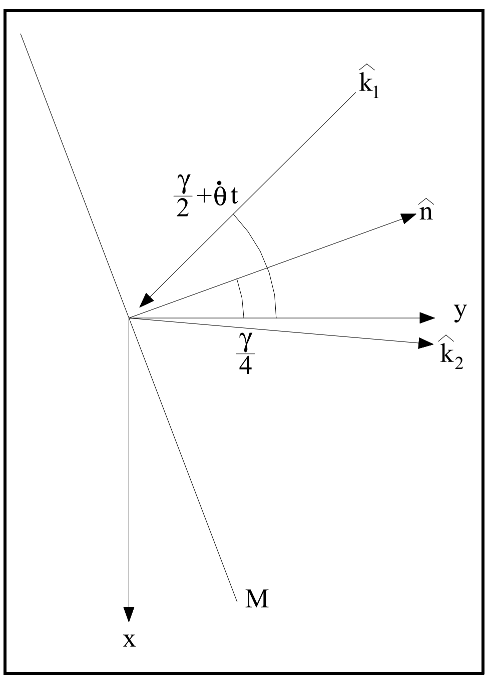

Figure 2. Rays at compound mirror. M is the surface of one element of the compound mirror

The tilt considered here is a difference in the z-components of the normals of the two elements of the compound mirror, called here a relative "out of plane" tilt. The primary effect of such a tilt is to make the images of stars entering the two ports travel at different angles across the focal plane. A secondary effect is to make the images travel at different speeds on the focal plane. For reasonable tilts, the latter is insignificant.

Consider a beam entering the port to the right in Fig. 1 in the xy plane, with propagation vector $\hat{k}_1$ (which has unit length). It reflects at the corresponding element of the compound mirror, whereafter it has propagation vector $\hat{k}_2$ (Fig. 2). Take the mirror normal to be tilted by an angle ε. It is

$$\hat{n} = \frac{1}{\sqrt{1+\varepsilon^2}} \begin{pmatrix} -\sin\frac{\gamma}{4} \\ \cos\frac{\gamma}{4} \\ \varepsilon \end{pmatrix} . \tag{1}$$

The input propagation vector is

$$\hat{k}_1 = \begin{pmatrix} \sin(\frac{\gamma}{2} + \dot{\theta} t) \\ -\cos(\frac{\gamma}{2} + \dot{\theta} t) \\ 0 \end{pmatrix} , \tag{2}$$

where $\dot{\theta}$ is the instrument rotation rate, and t=0 when the star is in the center of the field of view. The reflected ray $\hat{k}_2$ is given by

$$\hat{k}_2 = \hat{k}_1 - 2(\hat{k}_1 \cdot \hat{n})\,\hat{n} . \tag{3}$$

With the aid of Maple, using the Matlab Extended Symbolic Toolbox interface, this becomes[1]

$$\hat{k}_2 = \frac{1}{1+\varepsilon^2} \begin{pmatrix} \sin(\dot{\theta} t) + \varepsilon^2 \sin(\frac{\gamma}{2} + \dot{\theta} t) \\ \cos(\dot{\theta} t) - \varepsilon^2 \cos(\frac{\gamma}{2} + \dot{\theta} t) \\ 2\varepsilon \cos(\frac{\gamma}{4} + \dot{\theta} t) \end{pmatrix} \tag{4}$$

Taking the time derivative,

$$\frac{d\hat{k}_2}{dt} = \frac{\dot{\theta}}{1+\varepsilon^2} \begin{pmatrix} \cos(\dot{\theta} t) + \varepsilon^2 \cos(\frac{\gamma}{2} + \dot{\theta} t) \\ -\sin(\dot{\theta} t) + \varepsilon^2 \sin(\frac{\gamma}{2} + \dot{\theta} t) \\ -2\varepsilon \sin(\frac{\gamma}{4} + \dot{\theta} t) \end{pmatrix} \tag{5}$$

(As a check on the computer algebra, $|\hat{k}_2| = 1$, and $|d\hat{k}_2/dt| = \dot{\theta}$.) The angle of trajectories on the focal plane is

[1] This information is for technical communication only and does not constitute an endorsement of these products.

$$\varphi = \frac{\left(\frac{d\hat{k}_2}{dt}\right)_z}{\left(\frac{d\hat{k}_2}{dt}\right)_x} = -2\varepsilon \frac{\sin(\frac{\gamma}{4} + \dot{\theta} t)}{\cos(\dot{\theta} t) + \varepsilon^2 \cos(\frac{\gamma}{2} + \dot{\theta} t)} \qquad \textbf{(6)}$$

A relative tilt, ε, of 10 arcsec, which has been Lockheed's nominal specification [Huff, 2000], results in a relative angle of the trajectories of images of stars through the two ports of 0.14 pixel per 4096, which will result in a negligible contribution to the alignment budget.

The velocity of the images is modified by a factor $1 + \varepsilon^2 \cos(\gamma/2)$ (at the field center), resulting in a position difference, after TDI of 4096 pixels, of 7×10^{-6} pixel = 1.5 μarcsec. This is equivalent to a (static) change of the basic angle, and is utterly negligible.

Reference

Huff 2000 E-mail from L. Huff, Lockheed-Martin Corp., 13 Nov. 2000.

Distribution

FAME project, *via* website.