Global Fitting Overview

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Outline

• The objective of the Global Fitting is to interconnect the observing-spiral rotation models to yield single spacecraft rotation model.
• This step is to establish a globally consistent reference frame for the position and proper motions, and a corresponding set of data for the satellite attitude and geometric calibration of the instrument.
• This will be the most difficult stage in the reduction because of large number of parameters to be estimated simultaneously.
• The analysis tool is the weighted-least-square (WLS) fitting.
• A single WLS estimator is to yield all of the grid star parameters and all of the instrument parameters.
• Input will be a set of spiral parameters from the S/C rotation model resulted from the Spiral Reduction and some instrument parameters including optics and CCD from the centroid step.
• Output will be astrometric (position, proper motion and parallax) parameters each grid star and rigid spiral parameters (3 Euler angles) each spiral.

Coordinate Systems to be used

1. Detector reference system, pixel coordinate (τ,u,v)
2. Instrument ref. sys., (t,x,y,z)
3. Spacecraft(satellite) ref. sys. - (t,x,y,z)
4. ICRS, (T,X,Y,Z)

note: A reference to the celestial coordinates is eventually needed, but the ecliptic coordinates (λ, β) will be a working celestial coordinate, and the ecliptic is taken to be the fundamental plane.

Process

Observation Model → Observation Equation → Build Design Matrix → Solve Parameters → Improve the precision by iteration

Observation Model

A description of the observations in terms of the unknown parameters and other data. The instrument, attitude and astrometric models will be included.

Instrument Model

The point-spread function P_n varies with time, transverse pixel coordinate (v) and the color index for given CCD number n. This will be given from the centroid step.

The geometric instrument model specifies the relation between the Detector RS (τ,u,v) and the Instrument RS (t,x,y,z).

on-board time (τ) vs. S/C time (t)
For a given channel number n, an on-board time scale τ_{un} is to be associated with each sample. (u is a along-scan pixel coordinate of the centroid.) The corresponding time in the S/C ref. system, t_{un}, is known by means of the calibrated relation $t(\tau)$.

Field-to-Focal Plane Transformation
Let (ξ, η) be field angles. The mapping from the x-scan field angle to the x-scan pixel coordinate is:

$$v = M(\eta)$$

where M may be a low-order polynomial.

The location of the fiducial line ξ

$$\xi = K(\eta) + \Delta\xi_{mn}$$

where K may be a low-order polynomial, while $\Delta\xi_{mn}$ is the displacement of pixel column m relative to the mean position.

The basic angle γ

$$\cos\gamma = \mathbf{f_{-1}} \cdot \mathbf{f_{+1}}$$

where $\mathbf{f_{+1}}$ is the preceding field of view and $\mathbf{f_{-1}}$ the following field of view.

The instrument model is described by a number of parameters in P_n, M, K, $\Delta\xi_{mn}$ and γ. An assumption for the data analysis is that the instrument changes are slow enough that the parameters can be calibrated as functions of time.

Attitude Model

The attitude specifies the orientation of the instrument axes connecting the s/c ref. system to the ICRS. At any instant, the FAME orientation will be given by three Euler angles (φ, ψ, θ) and these angles can be modeled as continuous functions of time by means of polynomials, trigonometric functions or splines.

Astrometric Model

For a single star, the vector $\mathbf{X}(T)$ represents the position of the star at the epoch of observation T, wrt to the solar system barycenter.

$$\mathbf{X}(T) = \mathbf{X}_0 + \dot{\mathbf{X}}\ (T - T_0)$$

where the position vector $\mathbf{X}_0 = \mathbf{X}(T_0)$ of the star at the catalog epoch T_0 wrt the solar system barycenter. $\mathbf{X}(T_0)$ and $\dot{\mathbf{X}}$ are parameterized by the standard five astrometric parameters (α, δ, π, μ_α, μ_δ) and the radial velocity (V_R).

Observation Equation

Expression of the difference between observed and calculated observation in terms of different sources of error. This is a process to build design matrix.

Design Matrix

Suppose we have a system for FAME like,
F(x)+ε=b
where F=observation equation
ε=errors
b=N observable values
x=M parameters to fit

linearize for i^{th} observation by

$$F_i(x) = F_i(x_0) + ... \sum_j \frac{\partial F_i}{\partial x_j} x_j$$

let

$$\frac{\partial F_i}{\partial x_j} = A_{ij}$$

then, by a matrix form

$$Ax = b$$

where A, known as the design matrix of the LSQ problem, is a sparse matrix.

$\Rightarrow$ LSQ problem minimizing $||Ax - b||_2$ over x.

Solution Strategy

Solve the LSQ problem by the method of normal equations.
i.e., $A^T Ax = A^T b \Rightarrow Nx = y$ ($A^T A = N$, $A^T b = y$)

If the matrix N is *symmetric, positive definite and constructed properly*
$\Rightarrow$ block Cholesky decomposition: most efficient way to solve the normal eqn.

The first process will produce the design matrix selecting only the stars to be included in the process by means of an initial quality star test, and also prepare the data for the following iterations. The design matrix for the instrument parameters will be used to calculate astrometric parameters for grid stars. The second will perform one iteration for each activation, test the convergence of the process and, if a stopping criterion occurs, produce the final results.

Initially two astrometric parameters (positions for each star) and rotation parameters will be included and later five astrometric parameters will be calculated. The covariance matrix of the LSQ will be off-diagonal portion.

Two different sets of variables will be used: one set represent star parameters including the astrometric parameters and the other for non-star parameters including the FAME attitude and instrument parameters. The normal matrix of the system of equations is divided into four different parts N_{AA}, N_{AB}, N_{BA} and N_{BB} such as

$$\begin{vmatrix} \mathrm{N}_{AA} & \mathrm{N}_{AB} \\ \mathrm{N}_{BA} & \mathrm{N}_{BB} \end{vmatrix}$$

where A is a collection of coefficients for the astrometric parameters and B is the coefficients for the non-star parameters. N_{AA} is block diagonal with a block size of at least 5 and N_{BB} is block diagonal with a block size of number of non-star parameters each spiral.